Quantitative Macroeconomics

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Homework 3, due Thursday Oct 10 at 8.30am

Question 1. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \; (z \; h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1 - \delta) \; k_t \tag{4}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

(a) Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.

(b) Double permanently the productiviy parameter z and solve for the new steady state.

(c) Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labor and output.

(d) Unexpected shocks. Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.

(e) Optional Bonus Question: Can taxes explain differences in the speed of transition to steady-state?

- Add a permanent consumption tax. Recompute the new steady state, and the transitions.
- Add a permanent capital tax. Recompute the new steady state, and the transitions.

(f) Optional Bonus Question: Boldrin, Christiano and Fisher (AER, 2001) and Christiano (Minn QR, 1989)

- What if preferences take the form of Boldrin, Christiano and Fisher (AER, 2001)? That is, abstracting from labor choice,

$$u(c) = \ln(c_t - bc_{t-1}). \tag{5}$$

Recompute the transition as posed in Question 1.

- What if preferences take the form of Christiano (Minn QR, 1989)? That is, abstracting from growth,

$$u(c) = \ln(c_t - \bar{c}) \tag{6}$$

Recompute the transition as posed in Question 1. Plot the differences in the time path of savings.

- Now, allow for growth, i.e., $z_t = z_0(1+\lambda_z)^t$, and replicate Christiano's Chart 1-4 for Japan, and extend the exercise to as many countries as you can (e.g. China, Taiwan, Korea, South Africa and Zambia). Get historical data for the U.K. (as long time series as you can), and replicate those Charts.

(g) Bonus Question: Labor Choice Allow for elastic labor supply. That is, let preferences be

$$u(c_t, 1-h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{7}$$

and recompute the transition as posed in Question 1.

Question 2. Multicountry model with free mobility of capital (not labor) and progressive labor income tax

We are interested in studying the respective general equilibrium (GE) of closed economies of each of these countries and a the GE of a union with free mobility of capital. Labor and goods are not mobile.

Environment This is a multicountry static model $\ell = \{1, 2\}$. Each country is populated by a heterogenous households that differ in their permanent productivity η, supply labor, $h \in [0, 1]$ and capital $k_\ell \in [0, \bar{k}_\ell]$, where $\bar{k}_\ell$ is the country-specific capital endowment, and takes prices as given. Each country produces a single good with a representative CRS firm that operates in competitive markets.

Household Agents are heterogeneous in their permanent labor productivity η and face uncertainty on their wage. After they are born (and they realize their η) they receive an idiosyncratic shock ε_ℓ^i with probability π. Labor income taxation is according to Feldestein (1969) tax function that we saw in class.

An agent with productivity η in country ℓ solves:

$$\max_{\{c_\ell, k_\ell \in [0,\bar{k}_\ell], h_\ell \in [0,1]\}} \left(\frac{(c_\ell)^{1-\sigma}}{1-\sigma} - \kappa \frac{(h_\ell)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right)$$

subject to

$$c = \lambda \left(w_\ell \left(h_\ell \right) \eta_\ell \right)^{1-\phi_\ell} + r_\ell k_\ell^\eta + r_{-\ell} \left(\bar{k}_\ell - k_\ell \right)$$

where foreign investment is $\left(\bar{k}_\ell - k_\ell \right)$.

Firm problem Each country has a single representative firm that solves:

$$\max_{\{K_\ell^d, H_\ell^d\}} Z\left(K_\ell^d\right)^{1-\theta}\left(H_\ell^d\right)^{\theta} - w_\ell H_\ell^d - r_\ell K_\ell^d$$

(2.1) Consider the case that each of these two countries are closed economies. Write (a) the equilibirum of a closed economy, (2) an algorithm to solve it and (3) solve the economy.

(2.2) (a) Write the equilibirum of the union economy, (b) the algorithm to solve it and (3) solve the economy for a given set of parameters. Notice that in the union economy market clearing is given by union capital markets and country-specific labor markets:

- Union capital markets:

$$K_t^d = \int_\ell k_\ell^d = \sum_\ell \int_\eta k_\ell d\eta = \int_\ell \bar{k}_\ell = \bar{K}_\ell$$

  Notice that the union capital supply and demand cannot exceed the union capital endowment.

- Country-specific labor market clearing:

$$H_\ell^d = \int_\eta \int_\eta \eta h_\ell, \quad \forall\, \ell$$

(2.3) Discuss how you would choose the optimal progoressive taxation of labor income for these economies? Write the planners problem and an algorithm to solve it.