Quantitative Macroeconomics

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Homework 3, due Thursday Sep 28

Question 1. Value Function Iteration

Consider a stationary economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c, h_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t, 1-h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \ (\ h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1-\delta)\ k_t \tag{4}$$

Set $\theta = .679, \beta = .988, \delta = .013$. Also, to start with, set $h_t = 1$, that is, labor is inelastically supplied. To compute the steady-state normalize output to one.

1. Pose the recursive formulation of the sequential problem without productivity shocks. Discretize the state space and the value function and solve for it under the computational variants listed below. In all these variants use the same initial guess for your value function.
   (a) Solve with brute force iterations of the value function. **Plot your value function.**
   (b) Iterations of the value function taking into account monotonicity of the optimal decision rule.
   (c) Iterations of the value function taking into account concavity of the value function.
   (d) Iterations of the value function taking into account local search on the decision rule.
   (e) Iterations of the value function taking into account both concavity of the value function and monotonicity of the decision rule
   (f) Use Howard's policy iterations waiting until converged to solve the problem. Start the policy iteration at three different iterations of the value function, and report the differences.
   (g) Use policy iterations with 5, 10, 20 and 50 steps in between policy reassessments.

   Report the time and number of iterations needed per variant and describe your results.
2. Redo item 1 adding a labor choice that is continuous. For this, set $\kappa = 5.24$ and $\nu = 2.0$.
3. Redo item 1 using a Chebyshev regression algorithm to approximate the value function. Alternatively, use cubic splines. Compare your results.
4. Add productivity shocks, z_t, to the program. These producitivity shocks follow a two-states Markov chain where the two values of the shock are 1.01 and $1/1.01$. Set the unconditional mean of z_t to one. Solve for the value function of this stochastic economy using VFI together with your preferred approximation method. **Plot your value function.**