Methods versus Substance: Measuring the Effects of Technology Shocks on Hours[☆]

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Abstract

Calibration and modern (Bayesian) estimation methods for a neoclassical stochastic growth model are applied to make the case that the implicit identification of key parameters, rather than quantitative methodologies per se, is responsible for empirical findings. For concreteness, the model is used to measure the contributions of technology shocks to the business cycle fluctuations of hours and output. Along the way, new insights are provided in the parameter identification associated with likelihood-based estimation, the sensitivity of likelihood-based estimation to the choice of structural shocks is assessed, and Bayesian model averaging is used to aggregate findings obtained from different DSGE model specifications.

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1. Introduction

Over the past three decades, quantitative macroeconomics has witnessed controversies about the role of formal econometric methods. While there is some broad consensus that the neoclassical stochastic growth model — potentially augmented by heterogeneity, various types of frictions, and different sources of idiosyncratic as well as aggregate uncertainty — provides a useful framework for substantive empirical work, there is less consensus on how such models should be parameterized in view of the available economic data. Following Kydland and Prescott (1982), many researchers calibrate dynamic stochastic general equilibrium (DSGE) models, whereas other researchers use formal econometric methods to parameterize DSGE models and study their quantitative implications. The methodological controversies between the calibration and the estimation camp are summarized, for instance, in a Journal of Economic Perspectives symposium with contributions by Hansen and Heckman (1996), Kydland and Prescott (1996), and Sims (1996).

Despite the use of similar theoretical frameworks that are built around a stochastic growth model, quantitative results for specific macroeconomic questions reported in the literature often vary dramatically. Some of this variation is seemingly associated with the choice of empirical methods. For instance, estimates of the relative importance of technology shocks for fluctuations in hours worked range from less than 10% to more than 65%. Similar figures for output can be as low as 20% or as high as 100%. Among the numerous papers published on this topic, Hansen (1985)'s calibration of a stochastic growth model with indivisible labor yielded numbers at the high end, whereas the likelihood-based estimation of a DSGE model with New Keynesian frictions by Galí and Rabanal (2005) produced estimates at the low end.

In this paper, we make the case that it is not the choice of quantitative methodology (per se) that is responsible for empirical findings, but rather the implicit identification of key parameters associated with particular calibration or estimation approaches. Thus, sources of identification, and not a controversy over the use of formal statistical methods, should be at the center of the debate in quantitative macroeconomics. Rather than providing an abstract exposition, we consider a specific empirical application throughout the paper. The neoclassical stochastic growth model with neutral and investment-specific technology shocks and variable capital utilization is used to measure the contribution of the two types of technology shocks to business cycle movements of aggregate hours worked and output. Generalizations to applications that involve more elaborate DSGE models are discussed toward the end of the paper.

While building our case, we make three distinct contributions. First, the paper compares how key parameters are identified in a standard calibration versus a likelihood-based estimation. Since the identification in the context of a calibration of a stochastic growth model is well understood, a particularly novel aspect of our analysis is to provide an economic interpretation of the way the likelihood function extracts information about important model parameters. The second contribution is to document the sensitivity of likelihood-based estimates to the inclusion of additional shocks that complete the probabilistic structure of the DSGE model and implicitly affect the identification of key parameters. In our application, these additional shocks are non-technology shocks, such as exogenous preference or government spending processes. Finally, the third contribution is to use Bayesian model averaging to aggregate quantitative results from the estimation of DSGE models with different auxiliary shock specifications.

Starting with Canova and Sala (2009), the topic of identification has recently received particular attention in the DSGE model literature, as it has become apparent that estimation objective functions are often not very informative about important model parameters. Guerron-Quintana (2010) documents how parameter estimates of a variant of the Smets-Wouters model are affected by the choice of observables used in the Bayesian estimation. Iskrev (2010) and Komunjer and Ng (2009) develop formal conditions for the identifiability of DSGE model parameters based on first and second moments of model variables. The focus of our paper is to dissect the identification assumptions underlying typical calibration and likelihood-based estimation approaches.

To study how different empirical approaches achieve parameter identification, it is useful to distinguish steady-state-related parameters, parameters that characterize the law of motion of the exogenous shocks, and parameters that affect the endogenous propagation mechanism but not the steady state. While steady-state related parameters are typically identified from long-run averages, both in a calibration as well as in an estimation setting, the identification of the other parameters is more delicate. The stochastic growth model used in the empirical analysis requires us to identify the parameters of two technology shocks as well as an aggregate labor supply elasticity and the elasticity of capital utilization with respect to interest rate changes.

The investment-specific technology shock rotates a linear transformation curve between consumption and investment goods, and capital stock adjustments are assumed to be costless. In turn, this shock equals the relative price of investment, and thus its parameters are identifiable from observables without knowledge of the two elasticities. If capital utilization is assumed to be constant, the identification of the parameters associated with the neutral technology shock can be achieved conditional on steady-state-related parameters only. If, on the other hand, capital utilization is variable, the identification of the neutral technology shock parameters cannot be disentangled from the identification of one of the endogenous propagation parameters, namely, the interest rate elasticity of capital utilization.

While calibration approaches tend to use external information to identify the labor supply elasticity, e.g., from household level data, information about the composition of hours worked fluctuations into intensive and extensive margins, or balanced growth path restrictions, estimation approaches tend to extract information directly from the joint dynamics of output and hours worked. More specifically, in our application the DSGE model incorporates enough restrictions to identify technology shock innovations from the observables. We show how the relative movement of labor productivity and hours, that is, prices and quantities in the labor market, can identify the labor supply elasticity. In the variable capital utilization model, the utilization elasticity is identified from the shape of the impulse responses.

From an econometric perspective, the probabilistic structure of the DSGE model is incompletely specified in our application as well as many other applications. The baseline version of our DSGE model is purely driven by technology shocks and silent with respect to the nature of other shocks that contribute to aggregate fluctuations. Unlike the identification implicit in a typical calibration approach, the identification scheme associated with a likelihood-based estimation is sensitive to the completion of the probabilistic structure, which is typically achieved by introducing additional structural shocks or measurement errors. Thus, we systematically document the sensitivity of likelihood-based estimates to the inclusion of shocks that complete the probabilistic structure of the DSGE model. The tacit identification implications of auxiliary shocks have been a neglected topic and are likely to be very important for large-scale DSGE models that build on the work by Smets and Wouters

(2003, 2007).

More often than not, different methods of identifying key parameters of a DSGE model lead to different quantitative findings. This fragility is in part due to the restrictive —and in some dimensions misspecified— dynamic structure of the models and its inability to fit macroeconomic time series in all dimensions. In a calibration framework, this ambiguity can be communicated by reporting quantitative findings associated with the various plausible identification schemes. In an econometric framework, the relative fit of the various model specifications associated with different identification schemes can in principle be used to weight different quantitative results. This weighting can be coherently implemented in a Bayesian framework. Thus, we use model averaging to aggregate the findings from our estimation of DSGE models with different auxiliary shock specifications. Six model specifications that differ with respect to the non-technology shocks and whether or not we impose unit roots on the law of motion of the technology shocks are considered in the application.

The quantitative findings in the empirical application support our main methodological point. If the labor supply elasticity is set to 0.72 to match recent micro-level evidence provided by Heathcote et al. (2007), the fractions of hours and output fluctuations explained by technology shocks are 9% and 26%, respectively. If the elasticity is identified by the labor supply behavior along the intensive margin of prime age white males, the fractions drop to 1% and 20%. If the elasticity is determined by a sufficient condition on preferences to guarantee a balanced growth path, then the fractions increase to 29% and 33%. The estimation can generate a similar range of results, depending on the choice of data set and the shock specification. If the DSGE model is estimated based on observations for labor productivity, hours worked, the relative price of investment, as well as investment, the labor supply elasticity is around 0.1 which, translates into variance ratios of 1% for hours and 22% for output. If, on the other hand, the relative price of investment is excluded from the observables and a government spending shock rather than a preference shock is used to complete the probabilistic structure of the model, then the labor supply elasticity estimate increases to 1.56, and the variance ratios rise to 22% for hours and 33% for output. Thus, within each empirical methodology, differences among the explicit or implicit identification of key parameters can generate a wide spectrum of quantitative results, making a convincing case that quantitative macroeconomists from both the calibration as well as the estimation camp should place more emphasis on searching for reliable sources of identification of key parameters and making them transparent to their audiences.

The remainder of this paper is organized as follows. The stochastic growth model is presented in Section 2. We begin the empirical analysis with a restricted version of the model in which capital utilization is constant. Section 3 reviews identification schemes for the labor supply elasticity that are associated with standard calibration procedures. Section 4 reports on the DSGE model estimation using state-of-the-art Bayesian techniques. In Section 5 we allow for variable capital utilization. Recommendations for practitioners that generalize to other models and applications are provided in Section 6: first, determine the set of parameters that are most influential in shaping the quantitative findings; second, consider various plausible ways of identifying these parameters; and third, incorporate the identification schemes into your quantitative approach, be it a calibration or an econometric estimation of the DSGE model. Section 7. An Online Appendix provides detailed information on the data set, the implementation of the empirical analysis, and parameter estimates.

2. The Model Economy

We consider what we think is the latest implementation of the plain-vanilla real business cycles model: a stochastic growth model with two types of technology shocks and variable capital utilization. A neutral productivity shock affects total factor productivity. The second shock is investment-specific and shifts the slope of the transformation curve between consumption and capital goods. Our model is very similar to the one used by Fisher (2006). It is a simplified version of the model studied by Greenwood et al. (2000) in that we have only one type of capital.

The model economy is populated with a continuum of households with the following preferences:

$$\max_{\{C_t, X_t, H_t, K_{t+1}\}} \quad I\!\!E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \xi \, \frac{H_t^{1+1/\nu}}{1+1/\nu} \right) \right]. \tag{1}$$

Here, C_t denotes consumption and H_t hours worked. An appealing feature of these preferences is that the parameter ν is the Frisch elasticity of substitution of labor. β is, as always, the discount rate. ξ affects the marginal rate of substitution between consumption and leisure and determines steady-state hours. There is a constant returns to scale production function given by

$$C_t + \frac{1}{V_t} X_t = A_t (u_t K_t)^\alpha H_t^{1-\alpha}. \tag{2}$$

The left-hand side of (2) can be interpreted as a linear transformation curve between consumption and investment goods. The slope of this curve is shifted by the investment-specific technology disturbance V_t. The right-hand side takes the standard Cobb-Douglas form, A_t is an exogenous total factor productivity (or neutral) technology process, K_t is capital, and u_t is the capital utilization. Thus, the production technology is subject to two exogenous shocks: a standard neutral technology shock A_t and a sector-specific technology shock V_t. Capital depreciates geometrically at the utilization-dependent rate $\delta(u_t)$, yielding

$$K_{t+1} = (1 - \delta(u_t))K_t + X_t. \tag{3}$$

This economy is not distorted, and the welfare theorems allow us to solve the social planner's problem in order to find the equilibrium. Under competitive markets, the relative price of a unit of the investment good using the consumption good as numeraire, P_t, is equal to the reciprocal of the technology shock, $1/V_t$. Define $I_t = P_t X_t$, and hence the aggregate resource constraint can be written as

$$Y_t = C_t + I_t = A_t (u_t K_t)^\alpha H_t^{1-\alpha}.$$

The static Euler equation using prices is given by $H_t = \left(\frac{1}{\xi}\frac{W_t}{C_t}\right)^\nu$, while the dynamic Euler equation takes the form $1 = \beta \mathbb{E}_t \left[\frac{P_{t+1}/C_{t+1}}{P_t/C_t}\left((1-\delta(u_{t+1})) + u_{t+1}R_{t+1}\right)\right]$. The optimal choice of the utilization rate is given by $\delta'(u_t) = R_t$. In turn, the equilibrium wage equals the marginal product of labor $W_t = (1-\alpha)A_t(u_t K_t)^\alpha H_t^{-\alpha}$, while the rate of return satisfies $R_t P_t = \alpha A_t (u_t K_t)^{\alpha-1} H_t^{1-\alpha}$.

At this point, our model has two exogenous disturbances, namely, a neutral and an investment-specific technology process. To examine the effect of technology fluctuations on hours worked, we assume that

$$\begin{aligned}
(\ln A_t - \ln A_0 - \gamma_a t) &= \rho_{a,1}(\ln A_{t-1} - \ln A_0 - \gamma_a (t-1)) && (4)\\
&\quad + \rho_{a,2}(\ln A_{t-2} - \ln A_0 - \gamma_a (t-2)) + \sigma_a \epsilon_{a,t} \\
(\ln V_t - \ln V_0 - \gamma_v t) &= \rho_{v,1}(\ln V_{t-1} - \ln V_0 - \gamma_v (t-1)) && (5)\\
&\quad + \rho_{v,2}(\ln V_{t-2} - \ln V_0 - \gamma_v (t-2)) + \sigma_v \epsilon_{v,t}.
\end{aligned}$$

Thus, the log technologies fluctuate around linear deterministic trend paths, given by $\ln A_0 + \gamma_a t$ and $\ln V_0 + \gamma_v t$, respectively. If the autoregressive coefficients sum to one, the fluctuations are non-stationary and the technology processes can be rewritten as AR(1) processes in terms of growth rates. The most widely used specifications for the neutral technology process can be easily obtained as special cases of (4). If $0 \leq \rho_{a,1} < 1$ and $\rho_{a,2} = 0$, then technology follows a stationary AR(1) process. If $\rho_{a,1} + \rho_{a,2} = 1$, then technology has a unit root and the serial correlation of its growth rates is $-\rho_{a,2}$, which is often assumed to be zero. In order to restrict the autoregressive processes in (4) and (5) to trend stationarity, it is convenient to reparameterize them in terms of partial autocorrelations ψ_1 and ψ_2. Omitting the a and v subscripts, we let

$$\rho_1 = \psi_1(1 - \psi_2), \quad \rho_2 = \psi_2. \tag{6}$$

In case of a unit root, $\psi_1 = 1$. The analysis in this paper is conducted under the assumption that the two innovations $\epsilon_{a,t}$ and $\epsilon_{v,t}$ are normally distributed with zero mean and unit variance. Moreover, we assume that they are uncorrelated at all leads and lags.[3]

Regardless of whether the technology shocks have a stochastic trend component (unit root) or are trend stationary, the following transformations generate stationary variables:

$$\frac{Y_t}{Q_t}, \quad \frac{C_t}{Q_t}, \quad \frac{I_t}{Q_t}, \quad \frac{X_t}{Q_t V_t}, \quad \frac{K_{t+1}}{Q_t V_t}, \quad \frac{W_t}{Q_t}, \quad \text{where} \quad Q_t = A_t^{\frac{1}{1-\alpha}} V_t^{\frac{\alpha}{1-\alpha}}.$$

To approximate the model dynamics, we rewrite the equilibrium conditions in terms of these detrended variables, derive a non-stochastic steady state, log-linearize the equilibrium conditions around the steady state, and use a standard procedure to solve the resulting linear rational expectations system. For this extremely simple economy, a log-linear approximation is typically used in the literature because it is deemed to be accurate enough.

Let R^* denote the steady-state rental rate of capital. We use a depreciation function of the form $\delta(u_t) = \delta_0 + \delta_1(u_t^{1+1/\zeta} - 1)$ and assume that $\delta_1 = R^*/(1 + 1/\zeta)$ such that the steady state utilization rate equals $u^* = 1$. The parameters of the model economy belong to three categories. First, the parameters that affect the steady state of the model are the capital share of output α, the discount rate β, the steady-state depreciation rate δ_0,

[3] In principle, one could model $\ln A_t$ and $\ln V_t$ as vector autoregressive processes. However, empirically it turns out that the cross-correlations are small. Since the extension to a VAR process seems neither quantitatively important nor essential to the methodological issues discussed in this paper, we decided to impose that the two technology shocks are uncorrelated.

and the relative weight of consumption and leisure in the utility function ξ. Second, the parameters $\ln A_0$, $\psi_{a,1}$, $\psi_{a,2}$, σ_a, $\ln V_0$, $\psi_{v,1}$, $\psi_{v,2}$, and σ_v only affect the law of motion of the technology disturbances. The third category consists of parameters that only affect non-steady-state behavior, namely, the Frisch labor supply elasticity ν and the elasticity ζ of capital utilization with respect to interest rates. If $\zeta = 0$, then our model reduces to one with fixed capital utilization, the case we discuss in Sections 3 and 4. The extension to variable capital utilization, $\zeta > 0$, is subsequently provided in Section 5.

3. Calibration

This section summarizes the identifying information that is used in a typical calibration of a stochastic growth model. We start by discussing the steady-state related parameters (Section 3.1), and then we examine the identification of the exogenous shocks (Section 3.2) and the choices of the most relevant endogenous propagation (Section 3.3). As mentioned above, capital utilization is initially restricted to be constant, leaving the labor supply elasticity ν as the only parameter that affects the endogenous propagation but not the steady state of the model. Once we have determined values for all model parameters, we simulate output and hours worked data from the log-linearized model economy. We then compute variance ratios for HP-filtered (with the smoothing parameter set to 1600) simulated and actual data. Unless otherwise noted, our analysis is based on data from 1955:Q3 to 2006:Q4. Our findings are reported in Section 3.4. Precise data definitions are provided in the Online Appendix.

3.1. Steady-State-Related Parameters

Long-run averages of macroeconomic time series can and typically have been used in calibration studies to identify steady-state-related parameters. According to our specification, factor markets are competitive, and the aggregate production function has a Cobb-Douglas form. Hence, the implied labor share W_tH_t/Y_t is equal to $1 - \alpha$. While the observed labor share is time-varying (see, for instance, Ríos-Rull and Santaeulàlia-Llopis (2010)), it displays no clear trend. Hence, we target the steady state of the model to have the average labor share in the data, which we take to be 0.66. We also target an investment-to-output ratio of 28% and a yearly interest rate of 4%. These choices yield values of $\alpha = 0.34$, $\beta = 0.99$, and

$\delta_0 = 0.013$ in quarterly terms.[4] The value of the parameter ξ determines the steady-state level of hours worked and leisure. It could be set to match either the average level of hours worked per person in U.S. data or the fraction of total available time within a period devoted to hours worked. Parameter values could be obtained either from aggregate data on hours worked or from micro-level data from a time-use survey. In our specification, ξ does not affect the decision rules in a log-linear approximation and is irrelevant for the behavior of the model.

3.2. Parameters of the Shock Processes

The identification of the parameters associated with the law of motion of the exogenous shock processes is best understood in two steps. The first step is the construction of an empirical measure of the exogenous disturbances. It turns out that in the version of the DSGE model with fixed capital utilization, the construction of such measures only involves the (previously identified) steady-state-related parameters, but not the endogenous propagation parameters. The second step consists of determining the parameters of the shock processes based on the measured structural disturbances.

Measures of the Shock Processes. In the model economy, the investment-specific technology shock is equivalent to the (reciprocal of the) relative price of investment in terms of consumption. We construct this relative price by combining a price index for quality-adjusted equipment investment with a price index for investment in structures. With regard to the equipment investment price index, Gordon (1990), Greenwood et al. (1997), and Cummins and Violante (2002) reveal substantial evidence of biases in the trend of official price indexes due to the lack of quality adjustment. We build on the annual series of Cummins and Violante (2002) to construct our quarterly series of quality-adjusted equipment investment. Quarterly movements are imputed based on the official index reported by the Bureau of Economic Analysis (BEA) in the Fixed Asset Tables (FAT-BEA). As a price index for investment in structures we use the consumption deflator, P_t^C. The two indices are combined with a Tornquist aggregator to obtain a quality-adjusted price index for total investment,

[4] The definition of labor share has many subtleties that we avoid here altogether; see the aforementioned Ríos-Rull and Santaeulàlia-Llopis (2010) or Cooley and Prescott (1995). The three parameter values also determined the capital-to-output ratio, which is sometimes used for calibration instead of the investment-to-output ratio. For our model specification, capital measured in consumption terms is 2.54 times yearly output.

P_t^I. We then define $P_t = P_t^I / P_t^C$ and $V_t = 1/P_t$, normalizing the index such that $P_0 = 1$ in 1947.

The series for the neutral technology process A_t is computed using measures of per capita real output Y_t, capital K_t, labor input H_t, and an estimate of the capital share α.[5] What is non-standard in our analysis is that we have to construct a quality-adjusted capital stock.[6] To do so, we begin by generating a quarterly series for investment in efficiency units $X_t = (I_t^E + I_t^S)/P_t^I$, where I_t^E and I_t^S are total nominal investment in equipment and structures, respectively, and P_t^I is the quality-adjusted price index mentioned above. The quality-adjusted capital stock is obtained by the perpetual inventory method with the average physical depreciation rate of total capital calculated by Cummins and Violante (2002). The initial capital stock K_0 is calibrated using the observed level of output and the investment-to-output ratio in 1947.

Parameterizing the Shock Processes. Based on the empirical measures of $\ln A_t$ and $\ln V_t$, it is straightforward to determine the coefficients for the autoregressive models (4) and (5). While typically simple least squares are used to estimate the coefficients of the AR(2) processes, for internal consistency within this paper, we employ Bayesian techniques. This makes the estimates directly comparable to those reported for the full DSGE model in Section 4. Bayesian estimates are obtained by combining a prior density $p(\theta)$ for the parameters of an econometric model, denoted by $\theta \in \Theta$, with the density of the data given the parameters $p(Y|\theta)$ and applying Bayes Theorem to derive the posterior density $p(\theta|Y)$.

As discussed in Section 2, the AR(2) shock processes are parameterized in terms of partial autocorrelations ψ_1 and ψ_2; see (6). These processes are trend stationary if $-1 < \psi_1, \psi_2 < 1$ and become difference stationary if $\psi_1 = 1$. We estimate the parameters subject to $0 \leq \psi_1 < 1$ (deterministic trend) and $\psi_1 = 1$ (stochastic trend). In the former case, we assume that the first-order partial autocorrelation has a Beta distribution with mean 0.95 and standard deviation of 0.02. For both the difference-stationary and trend-stationary specification, it is assumed that the second-order partial autocorrelation is uniformly distributed on the interval $(-1, 1)$. Our priors are fairly agnostic with respect to the average growth rate of

[5] See the Online Appendix for details.

[6] Note that A_t is not what is typically referred to as the Solow residual, where the same formula is used with a series of capital published by NIPA and the investment-specific technical change is largely not taken into account.

the technology processes and the location parameters $\ln A_0$ and $\ln V_0$, which determine the log levels of the technology disturbances. The priors for the innovation standard deviations are centered at 1% with a large variance. A summary is provided in the first five columns of Table 1.

We construct a joint likelihood function for the two technology processes based on a sample that ranges from 1955:Q3 to 2006:Q4, conditioning on observations from 1954:Q3 to 1955:Q2. Posterior means and 90% probability intervals are reported in the last four columns of Table 1. Both technology processes are highly persistent and the estimates of ψ_1 exceed 0.97. The growth rates of the neutral technology process are essentially uncorrelated, that is, $\hat{\psi}_{2,a}$ is near zero, whereas the growth rates of $\ln V_t$ are strongly serially correlated with $\hat{\psi}_{2,v} \approx 0.8$. It is interesting to note that the deterministic component of technology growth is solely due to $\ln V_t$, which implies that it is embodied in the physical capital stock.

3.3. Endogenous Propagation Parameters: The Labor Supply Elasticity

In order to identify the endogenous propagation parameters, a typical calibration does not use evidence from time series movements of endogenous variables that are centrally related to the question that is addressed. In the context of our application, which aims at measuring the fraction of cyclical variation in output and hours that is due to technology shocks, this would imply not to use the aggregate dynamics of output and hours as a source of information about the labor supply elasticity. We will in turn discuss a variety of alternative approaches that have been pursued in the literature to parameterize ν.

Direct Estimates of the Labor Supply Elasticity. There is a large literature that directly provides estimates of the Frisch elasticity based on the analysis of micro-level data.[7] These estimates are typically very small. In his survey paper, Pencavel (1986) reports that most estimates for men are between 0 and 0.45, with 0.2 being a typical point estimate.[8] The labor supply elasticity is typically measured based on information about the intensive margin of prime age white males, who are full-time workers in most periods. However, it is well documented that a large fraction of hours fluctuations is accounted for by movements in and out of employment (see, for instance, Kydland and Prescott (1991), who describe the

[7] MaCurdy (1981), Altonji (1986), and Browning et al. (1985), to name a few classic papers.
[8] For a more recent survey, see Keane (2010).

extensive margin as responsible for two-thirds of the variation), and that those that have the most hours variation along the cycle are not prime age males (see Kydland (1984), Ríos-Rull (1993), and Kydland and Prescott (1988)). In addition, Low (2005) and Domeij and Floden (2006) argue that, due to borrowing constraints, the estimates may be biased downward (the latter by up to 50%). Imai and Keane (2004) estimate the elasticity explicitly accounting for human capital accumulation and obtain a value as high as 3.82. With human capital accumulation of the learning-by-doing variety, the macro models would need to be adjusted as in Chang et al. (2002). More recently, Heathcote et al. (2007) estimate a value for the household elasticity of 0.72 using a definition of the household that includes both a husband and a wife, while Heathcote et al. (2008) obtain 0.38 when taking into account household heterogeneity and measurement error but without using the notion of a multi-person household.

Information on the Composition of Hours Fluctuations. Most of the variation in hours over the business cycle occurs along the extensive margin (number of workers rather than hours per worker). To account for such variation, Rogerson (1988) developed a model where agents care about leisure but face a non-convexity in the opportunities to work. These agents can use a lottery arrangement to maximize the utility ex ante. Ex post some will work and others will not. After aggregation, this arrangement yields quasi-linear preferences for a representative consumer, which corresponds to $\nu = \infty$ in our model. Hansen (1985) used these lottery arrangements as the source of his calibration strategy in a seminal paper and not surprisingly found that hours move a lot in response to productivity shocks. Chang and Kim (2006) in multi-person households and Rogerson and Wallenius (2010) explicitly considering retirement have recently revisited the role played by non-convexities without using lotteries. Both papers argue that the macro elasticity, the elasticity that applies to an aggregate model with a stand-in household, is much larger than the elasticity that arises from studies using micro data, and advocate a value for the aggregate Frisch elasticity of around 1.

Turning the Labor Supply Elasticity into a Steady-State-Related Parameter. Functional-form restrictions on the representative household's preferences have been used to identify the labor supply elasticity based on long-run growth patterns. A reasonable description of the last 100 years of Western experience is the statement that while there has been a massive increase of wages, by an order of magnitude if not more, interest rates and the allocation of hours per capita have not displayed any such long-run trend. Most preference structures are not consistent with this pattern. Business cycles research typically uses utility functions that are consistent with this long-term behavior. The most popular among them are Cobb-Douglas utilities, $(c^\gamma \ell^{1-\gamma})^{1-\sigma}/(1-\sigma)$, where $\ell = 1-H$ is leisure and the endowment of time is normalized to one.[9] For this utility function, the Frisch elasticity depends only on the steady-state allocation of hours worked, H^*, via the expression $\nu = (1 - H^*)/H^*$. Another type of preference structure used in the literature that is consistent with a balanced growth path and where the steady-state level of hours also determines the Frisch elasticity of substitution is posed by King et al. (1988). The balanced growth path requirement by itself does not restrict the value of the Frisch elasticity, which in fact can be anything, provided that one chooses a per-period utility function that is rich enough.[10] However, the choice of utility function based on sufficient conditions for a balanced growth path ends up implying a value for the Frisch elasticity based on steady-state considerations that have really nothing to do with the willingness of households to substitute hours worked across time. If the average hours worked per adult per week is set to one-third,[11] which is a standard choice, the implied elasticity is 2. A choice of 25 hours per week that weighs young people and retirees more heavily yields an elasticity of 3. These values are much larger than those arising from the study of actual households.

Summary. We have considered various methods of identifying the aggregate labor supply elasticity ν. It turned out that these methods yielded a large range of values, which is at least in part due to the fact that the DSGE model is quite stylized. However, even more complicated DSGE models that have been studied in the literature are unable to reconcile the evidence from the various sources of information considered above. Since there is disagreement among researchers with respect to the reliability of the various identification approaches, we will compute variance ratios for five different choices. $\nu = 0.2$ represents micro-level estimates based on labor adjustments of males along the intensive margin, while $\nu = 0.72$ is a micro-level estimate that takes to heart some of the criticisms of macroe-

[9] This was essentially the choice of Kydland and Prescott (1982).

[10] Consider, for example the preferences given by $\frac{C^{1-\sigma}[1+(\sigma-1)\xi H^{1+\frac{1}{\nu}}]^{\sigma}-1}{1-\sigma}$. Here the Frisch elasticity is ν and there is a balanced growth path. See Shimer (2009) and Trabandt and Uhlig (2009).

[11] The U.S. Bureau of Labor Statistics reports (http://www.bls.gov/opub/working/page17b.htm) 67 hours per week worked by a married couple between the ages of 25 and 54. Conventionally, we think of 100 hours per week per person as the available discretionary time.

conomists about what is the object of interest when measuring the aggregate labor supply elasticity. We consider $\nu = 1$ as an attempt to capture the macro elasticities that arise from non-convexities (but not lotteries) argued by Chang and Kim (2006) and Rogerson and Wallenius (2010). Finally, we look at the high value of $\nu = 2$ found as the value arising from a sufficient condition to get a balanced growth path.

3.4. Relative Importance of Technology Shocks

We are now in a position to examine the answers obtained from the different calibration strategies. We simulate the linearized DSGE model for 310 periods using only the neutral technology process (A), only the investment-specific technology process (V), and both technology disturbances (A+V). We discard the first 100 observations and calculate the variances of HP-filtered output and hours based on the remaining 210 observations. These variances are then divided by the variances of HP-filtered postwar U.S. aggregate output and hours worked. The simulation steps are repeated 1,000 times, and the entries in Table 2 correspond to the means and standard deviations of the variance ratios across the 1,000 simulations.

As is well known, e.g., King and Rebelo (1999), the variance ratios for hours worked are very sensitive to the choice of labor supply elasticity. Our values range from 0.01 to 0.32 for the two technology shocks combined. If we take the Frisch elasticities estimated by Heathcote et al. (2007) ($\nu = 0.72$) using households' total hours variation, the variance ratio is 0.09. An elasticity based on males' responses along the intensive margin ($\nu = 0.2$) yields essentially no contribution of productivity shocks to movements in hours. The elasticity based on the Cobb-Douglas utility function and one-third of the time allocated to work ($\nu = 2$) yields a variance ratio of 0.32.[12] A labor supply elasticity of $\nu = 1$ that is consistent with aggregate fluctuations generated by a heterogeneous agent economy with indivisible labor supply results in a variance ratio for hours of 0.14. Interestingly, the investment-specific technical change is the main culprit of hours variation with over 60% of the total variation in all model economies.

The variance ratios for output are also increasing in ν and range from 20% to 37%. Output variation is directly affected by the neutral technology shock and indirectly through

[12] A similar value was obtained by Kydland and Prescott (1991). They reported a ratio of standard deviations of about two-thirds, which corresponds to approximately 0.44 in terms of variances.

the fluctuation of hours and capital over the business cycle. We find that output variation is mostly driven by the neutral technology process.[13] Finally, the results are somewhat sensitive to the parameterization of the shock processes. The contribution of technology shocks is larger for the deterministic trend than for the unit root specification.

4. Bayesian Estimation of the DSGE Model

We now turn to formal econometric methods to determine the importance of productivity shocks for movements in hours. We use Bayesian estimation techniques, reviewed in An and Schorfheide (2007), partly as a result of personal tastes and expertise and partly because they reflect state-of-the-art econometrics and are by now widely used in practice. In Section 4.1 we describe the data set used for the benchmark estimation of the DSGE model and discuss how the probabilistic structure of the DSGE model is completed to make it amenable to estimation. Priors and posteriors for the DSGE model parameters and the relative importance of technology shocks are presented in Section 4.2. Section 4.3 contains a detailed analysis of how the labor supply elasticity is identified through the likelihood function. Finally, Section 4.4 focuses on the second and third contributions of this paper: we examine how the estimation of the labor supply elasticity is affected by auxiliary assumptions about non-technology shocks and use Bayesian model averaging to weight competing model specifications. Moreover, we also discuss the sensitivity of the inference to the choice of observables.

4.1. Benchmark Data Choice and Model Completion

A crucial step in the estimation is the choice of observables Y that enter the likelihood function $p(Y|\theta)$. Here, θ stacks the DSGE model parameters that appeared in Section 2. Since the goal is to determine the contribution of productivity shocks to the variation of hours and output, these series should be ingredients for our estimation. Following the tradition in econometrics, we use labor productivity instead of total factor productivity, because the construction of the latter would require the knowledge of parameters that we are trying to

[13] One should not compare the contribution of the neutral shock with the total factor productivity (TFP) shock in most of the literature because they are computed very differently. In fact, the procedure that we followed in this paper reduces the role of the neutral technology shock because we increase the size of the stock of capital.

estimate. Fortunately, according to our theory the investment-specific technology shock is under perfect competition exactly the relative price of consumption versus investment. This gives us the three main series to use in the estimation: the log levels of labor productivity Y/H, hours worked H, and relative price of investment goods P.

With three series and two shocks, there exists a linear combination of the three series that can be predicted without error conditional on past observations. To overcome the singularity, we add an additional structural shock that is able to contribute to the business cycle fluctuations of hours and output. Given the findings of, among many others, Hall (1997) and Chari et al. (2007), a natural additional shock is a preference shock that affects the choice of hours worked. Consider the following variation of the utility function:

$$U_t = \ln C_t - B_t \xi \frac{H_t^{1+1/\nu}}{1+1/\nu}, \tag{7}$$

where $\ln B_t = \rho_b \ln B_{t-1} - (\sigma_b/\nu)\epsilon_{b,t}$ is such a preference shock. This will be our benchmark specification.

4.2. From Priors to Posteriors

Priors. The joint prior distribution for the DSGE model parameters is constructed as a product of marginal distributions, which are summarized in Table 3. It is useful to distinguish the same three groups of parameters as we dud previously in the calibration analysis. The prior elicitation for the steady-state related parameters α, β, and δ_0 uses the same sources of information as in Section 3.1. This elicitation is based on long-run averages of observables that are not included in the likelihood function, such as real interest rates, the labor share, and the size of the capital stock.[14] We use degenerate priors for two of the parameters: the discount factor and the depreciation rate are fixed at $\beta = 0.99$ and $\delta_0 = 0.013$. Based on the labor share data, we choose a prior for α that is centered at 0.34 with a standard deviation of 0.02.

The remaining steady-state related parameters are $\ln A_0$, $\ln V_0$, ξ, as well as the technology growth rates γ_a and γ_v. In order to simplify the prior elicitation, we use a re-parameterization that replaces $\ln A_0$ by $\ln Y_0$ and ξ by $\ln H^*$. As in the estimation of the technology shock parameters based on the measured processes in Section 3.2, the priors for

[14]Del Negro and Schorfheide (2008) show how to automate this kind of prior elicitation.

these parameters are fairly diffuse. The prior distributions for the partial autocorrelations and the innovation standard deviations of the two technology processes are identical to those reported in Table 1. The prior distribution for the autocorrelation of the preference shock is centered at 0.95 and has a standard deviation of 0.02. Finally, a prior for the endogenous propagation parameter, that is, the Frisch labor supply elasticity, needs to be specified. Our prior for the Frisch labor supply elasticity is centered at the balanced growth path value of $\nu = 2$, but with a standard deviation of one. Hence, a 90% a priori credible interval encompasses values found in studies that use micro-level data for employed males, as well as the values necessary to be able to explain most of the observed volatility in hours worked in a stochastic growth model driven by technology shocks.

Posteriors. The DSGE model is estimated based on observations from 1955:Q3 to 2006:Q4, conditioning on observations from 1954:Q3 to 1955:Q2. This conditioning will allow a subsequent comparison of marginal likelihoods between the DSGE model and a VAR. We use the Markov-Chain Monte Carlo methods reviewed in An and Schorfheide (2007) to obtain draws from the posterior distribution of the DSGE model parameters. The estimates of the technology shock parameters are very similar to those reported in Table 1. In part due to the fairly tight prior on α, the posterior mean estimates fall in the range of 0.33 to 0.40. Thus, the specific statistical methods used here did not change the estimates of these parameters. The coefficient estimates for the preference and government spending shocks are such that the model is able to capture the variation in output and hours worked that is not explained by the two technology shocks. Finally, we obtain an estimate of the Frisch elasticity of 0.85, which is a credible interval that ranges from 0.34 to 1.33. This estimate is in the range of those recently obtained by Heathcote et al. (2007, 2008) using micro-level data. A table with detailed estimation results is available in the Online Appendix.

Relative Importance of Technology Shocks. The neutral and investment-specific technology shocks combined account for about 11% of the variation in hours worked and 26% of the variation in output. Given the estimated labor supply elasticity of 0.85, the estimates of the relative importance of technology shocks for output and hours fluctuations are similar to those obtained from the calibration with $\nu = 0.72$.

4.3. Identification of the Labor Supply Elasticity

The likelihood function, at the heart of econometrics, peaks near parameter values for which the model-implied autocovariance function of the observables matches the sample autocovariance function as closely as possible in terms of a statistical metric. It does so by forcing each shock in the model to contribute particular autocovariance features, which in total have to mimic the sample autocovariances. This matching is, unfortunately, often difficult to interpret because there is no transparent link from patterns in the data to particular parameter estimates. The goal of this section is to shed some light on why our likelihood-based estimation yields a fairly low estimate of the labor supply elasticity.

Broadly speaking, technology shocks for which we can construct observations independently of the labor supply elasticity play the same role as exogenous demand shifters (or instrumental variables) in the analysis of traditional simultaneous equations systems: they perturb the market equilibrium and move prices and quantities. However, in DSGE models these technology shocks tend to shift both supply and demand. Since the slope of the labor demand function is essentially identified from the average labor share through functional form assumptions, observing the movements of wages and hours in response to the perturbation is sufficient for the determination of the labor supply elasticity.

In the remainder of this subsection, we first show that the labor supply elasticity is identifiable from the dynamic response of hours and wages, which in our model equal labor productivity, either to a neutral or an investment-specific technology shock. Second, we estimate a structural VAR identified with minimal restrictions on actual data as well as two simulated data sets. The simulated data sets are generated from the DSGE model parameterized with a small and a large value of ν. An impulse response comparison sheds light on whether the data favor a low or high labor supply elasticity. To ease the exposition, we assume that the technology shocks follow unit root processes by restricting $\psi_{1,a} = \psi_{1,v} = 1$.

Along the response to a technology shock, the labor supply condition, written in terms of temporal differences Δ, has to be satisfied:

$$\Delta\hat{h}_t = \nu(\Delta\hat{w}_t - \Delta\hat{c}_t). \tag{8}$$

Here, $\hat{h}_t$ denotes hours in percentage deviations from its steady state, and $\hat{w}_t$ and $\hat{c}_t$ denote percentage deviations of *detrended* real wages and consumption from their respective steady states. Since consumption is not included in the list of observables used in our likelihood-based estimation, we will replace it by a function of wages, hours, and technology shock. We show in the Online Appendix that in response to a one standard deviation investment-specific technology shock in period $t = 1$, the wage and hours dynamics for $t > 1$ can be expressed as

$$\Delta\hat{h}_t = \nu \left[\Delta\hat{w}_t + r^*(1-\alpha^{-1})\hat{w}_t - (-\psi_{2,v})^{t-1}\frac{\sigma_v}{1-\alpha} \right], \tag{9}$$

where $r^* = R^*/(R^* + 1 - \delta_0)$, and $R^* = e^{(\gamma_a+\gamma_v)/(1-\alpha)}/\beta - (1-\delta_0)$. A similar expression can also be obtained for the response to a neutral technology shock. Recall that r^* and α are identifiable from long-run averages of the labor share, real interest rates, and the investment-capital ratio, which enter our estimation objective function implicitly through the prior distribution. Thus, information on the impulse responses of wages (which are equal to average labor productivity in our Cobb-Douglas environment with constant factor shares) and hours worked suffices to identify the labor supply elasticity.

The DSGE model implies that data on the relative price of investment provide a direct observation on the investment-specific technology shock. Moreover, the assumption that this technology shock is exogenous generates an exclusion restriction that is sufficient to identify the dynamic responses of labor market variables to innovations in $\epsilon_{v,t}$. Thus, we deduce that the likelihood function contains identifying information on the labor supply elasticity.

In order to decode the identifying information, the following experiment is conducted. Using actual data and data simulated from the DSGE model, a structural VAR(4) in labor productivity growth, hours worked, and investment-specific technology growth is estimated. The innovations of the structural VAR are interpreted as innovations in the neutral technology shock, the investment-specific technology shock, and a non-technology shock. The structural VAR is identified by the following three restrictions that are also hardwired in the DSGE model. The first two restrictions are generated by the assumption that the investment-specific technology growth (reductions in the relative price of investment) is exogenous and follows an AR(1) process. Thus, the other two shocks have no effect on investment-specific technology. The third restriction is that the non-technology shock does not shift the labor demand schedule upon impact because capital is fixed in the short run.

The resulting VAR-based impulse responses are depicted in Figure 1. The figure shows 90% credible bands for responses estimated based on U.S. data as well as posterior mean responses based on data that have been generated from a DSGE model with Frisch elasticities $\nu = 0.2$ (solid) and $\nu = 2$ (dashed).[15] The hours responses provide conflicting information about the labor supply elasticity. To match the responses to an investment shock and the non-technology shock, the labor supply elasticity should be small; that is, a value of 0.2 is preferred over a value of 2. To reproduce the empirical response to a neutral technology shock, on the other hand, a large Frisch elasticity is needed. The overall estimate of ν is ultimately determined by the implicit weighting of the discrepancy between sample and DSGE model implied autocovariance functions (and hence VAR-based impulse response function estimates) encoded in the likelihood function. It turned out to be low, which suggests that the B- and V-shock responses received relatively more weight than the A-shock response.

4.4. Sensitivity of Estimation to Auxiliary Assumptions

The Bayesian estimation required us to make an auxiliary assumption about the business cycle fluctuations that are not caused by technology shocks. Moreover, we had to select a specific set of observables to construct the likelihood function. Since the identifying information in the likelihood function depends on both the data Y as well as the probabilistic structure of the exogenous shocks that ultimately generate the distribution of the data, we will now consider the robustness of our estimation to changes in the exogenous shocks as well as the set of observables.

Alternative Shock Specifications. Instead of a preference shock, one can introduce a demand shock, such as a government spending shock, which changes the aggregate resource constraint to $Y_t = C_t + I_t + G_t$. We assume that government expenditures are financed by lump-sum taxes and are determined as a time-varying fraction of total output, $G_t = (1 - 1/g_t)Y_t$. The process for government expenditures is exogenous and evolves according to $\ln(g_t/g^*) = \rho_g \ln(g_{t-1}/g^*) + \sigma_g \epsilon_{g,t}$.[16] Second, we consider a version of our model in which the preference

[15]The remaining parameters used to simulate the DSGE model are obtained by reestimating the DSGE model subject to the restrictions that $\nu = 0.2$ ($\nu = 2$) and $\psi_{1,a} = \psi_{1,v} = 1$.

[16]This specification leads to the relationship $\ln Y_t = \ln(C_t + I_t) + \ln g_t$ and implies that the government share of output is stationary.

shock and the government expenditure shock are simultaneously active. Thus, we denote the two additional model specifications by $\{A, V, G\}$ and $\{A, V, B, G\}$ and the benchmark specification of the previous subsection by $\{A, V, B\}$.[17] In order to estimate the model specifications with the government spending shock, we set $g^* = 1.2$ based on the average post-war government spending to GDP ratio. The prior distribution for ρ_g is centered at 0.95 and has a standard deviation of 0.02. Finally, we consider model specifications with difference stationary technology shocks, that is, $\psi_{1,a} = \psi_{2,a} = 1$. The location parameters of the Inverse Gamma priors for σ_b and σ_g (see Table 3), are chosen such that the priors for the various model specifications have similar implications with respect to the importance of non-technology shocks for the business cycle fluctuations of output and hours as well as for the volatility of output growth and hours. In particular, our priors imply that the 90% *a priori* credible intervals for the combined effect of the non-technology shock range from 0 to 25% for output and 10 to 100% for hours.

Alternative Data Sets. Since the accumulation of the quality-adjusted investment series provides a measure of the capital stock, which in combination with aggregate output and hours worked identifies the neutral technology shock via the production function (see Section 3.2), we consider the quality-adjusted investment, $\ln X_t$, as a fourth observable. In addition to the benchmark sample $\{Y/H, H, P\}$, we estimate the stochastic growth models on the samples $\{Y/H, H, X\}$ and $\{Y/H, H, P, X\}$, with the qualification that only the four-shock model is estimated based on the four-variable sample.[18]

Quantitative Results. Table 4 reports posterior means and 90% posterior credible intervals for the labor supply elasticity based on the various combinations of estimated model specifications and data sets. If the observables consist of productivity, hours worked, and the relative price of investment, the posterior mean estimates range from 0.30 to 0.85 and are somewhat larger if the unit root restrictions are not imposed. If the price of investment is replaced by the quantity, the estimates of ν tend to increase, in particular if the government spending shock is included as a shock in the model. This finding is at least qualitatively

[17] Modern variations on this model, containing additional frictions such as sticky prices and wages (and choices of households outside their labor supply function), monopolistic competition, monetary distortions, and the like, allow for many more shocks. See, for example, Smets and Wouters (2007).

[18] Guerron-Quintana (2010) examines how data choice affects parameter identifiability in the Smets-Wouters model by eliminating subsets of the seven macroeconomic time series that are typically used to estimate the Smets-Wouters model.

consistent with the impulse responses in Figure 1. The $\{Y/H, H, P\}$ data set contains direct information on the investment-specific technology shock. The figure indicates that the responses to an innovation of the investment-specific technology shock are better matched by a low labor supply elasticity. The $\{Y/H, H, X\}$ data set, on the other hand, in conjunction with the model-implied capital accumulation and production function, tends to identify the neutral technology shock and the responses to its innovation well. According to Figure 1, these responses are better matched with a high value of the labor supply elasticity. Unfortunately, we do not have a good explanation as to why the change in the labor supply elasticity estimate is most pronounced for the $\{A, V, G\}$ model. Finally, if the four-shock model specification is estimated based on four series, the labor supply elasticity estimates drop to 0.17 (unit root restrictions not imposed) and 0.12 (unit root restrictions imposed).

So far, the econometric analysis has generated multiple sets of parameter estimates, which in turn lead to a multitude of answers for our quantitative question. For each model specification and data set combination, Table 5 reports posterior means and standard deviations of variance ratios computed from HP-filtered simulated and actual data. All entries refer to the combined effect of neutral and investment-specific technology shocks on output and hours worked. The variance ratio results mimic the labor supply elasticity estimates: high elasticities yield large effects. The largest effect of technology shocks on hours worked fluctuations is obtained from the $\{A, V, G\}$ specifications estimated based on $\{Y/H, H, X\}$ data, explaining 20% of the observed variation when the shocks are restricted to follow unit root processes and 22% otherwise. The corresponding numbers for the output fluctuations are 32% and 33%, respectively. If, on the other hand, the DSGE model is estimated based on $\{Y/H, H, P, X\}$ observations, the labor supply elasticity is around 0.1, which translates into variance ratios of 1% for hours and 22% for output. Thus, the range of answers generated by estimating the DSGE model is about as wide as the range of results obtained by the various calibrations considered in Section 3.4. We deduce that within each empirical methodology, differences among the explicit or implicit identification of key parameters can generate a wide spectrum of quantitative results, making a convincing case that quantitative macroeconomists from both the calibration as well as the estimation camp should place more emphasis on searching for reliable sources of identification of key parameters and making them transparent to their audiences.

Model Averaging. Not all estimated model specifications track the time series data equally well. In a Bayesian econometric framework, it is natural to assign more weight to parameter estimates and predictions obtained from model specifications that attain a better time series fit. Formally, one can use log marginal likelihoods to update prior model probabilities.[19] For each of the three data sets, Table 6 reports log marginal likelihood differentials (or log Bayes factors), using the specification with the highest marginal likelihood as a benchmark. The log Bayes factors are converted into posterior model probabilities under the assumption that all DSGE model specifications have equal prior probability. For the $\{Y/H, H, P\}$ data set, it appears to be slightly preferable to impose unit roots in the two technology processes and to augment the technology-driven DSGE model with a government expenditure shock instead of a preference shock. Based on the $\{Y/H, H, X\}$ observations, the trend-stationary specifications are preferred in the three-shock models. However, the best fit is obtained by the four-shock version with unit root technology processes. Overall, the log marginal likelihood differentials are fairly small, indicating that the data can only imperfectly discriminate among the various specifications.

The boldfaced entries in Table 5 indicate specifications with posterior model probabilities higher than 3%. Weighted by posterior model probabilities, we conclude from the $\{Y/H, H, X\}$ data that 5% of hours fluctuations and 25% of output fluctuations can be explained by technology shocks. The corresponding numbers for the $\{Y/H, H, P\}$ data set are 2% and 21%, respectively. Due to the small value of the estimated labor supply elasticity based on the $\{Y/H, H, P, X\}$ data set, hours essentially do not move in response to technology shocks, and they explain 22% of output fluctuations.

5. Variable Capital Utilization

This section extends the previous analysis to the case of variable capital utilization ($\zeta > 0$). From a substantive perspective, variable capital utilization affects the propagation of structural shocks. The effective capital input is no longer pre-determined upon impact of the shock because firms can adjust capital along the utilization margin. Since the marginal

[19] Consider a collection of models $\mathcal{M}_m$, $m = 1, \ldots, M$. The marginal likelihood is defined as $p(Y|\mathcal{M}_m) = \int p(Y|\theta, \mathcal{M}_m)p(\theta|\mathcal{M}_m)d\theta$. The posterior probability of $\mathcal{M}_m$ is $\pi(\mathcal{M}_m|Y) = \pi(\mathcal{M}_m)p(Y|\mathcal{M}_m)/p(Y)$, where $p(Y) = \sum_{m=1}^{M} \pi(\mathcal{M}_m)p(Y|\mathcal{M}_m)$ and $\pi(\mathcal{M}_m)$ is the prior probability of $\mathcal{M}_m$. From a non-Bayesian perspective, the marginal likelihood provides a measure of in-sample fit that is adjusted by a penalty for model complexity.

product of labor rises as the effective capital input increases, hours tend to respond more strongly to technology shocks. Variable capital utilization is a mechanism that is present in many business cycle models to enhance their empirical performance; see the comprehensive discussions in Greenwood et al. (2000) and Christiano et al. (2005). From a methodological perspective, the variable utilization model includes an additional endogenous propagation parameter that needs to be identified. Moreover, it is not possible anymore to determine the parameters of the neutral technology shock independently of the endogenous propagation parameters, which is the case in many DSGE models. Section 5.1 reexamines the calibration analysis, and the variable capital utilization model is estimated in Section 5.2. It remains the case that within each empirical methodology, differences among the explicit or implicit identification of key parameters can generate a wide spectrum of quantitative results. Finally, Section 5.3 provides some discussion of results obtained from various other variants of the stochastic growth model that have been considered in the literature.

5.1. Calibration

Neither the calibration of the steady-state-related parameters, nor the calibration of the parameters associated with the investment-specific technology shock and the labor supply elasticity, is affected by the variable capital utilization. We only have to reconsider the calibration of the neutral technology shock parameters as well as the choice of the elasticity of capital utilization with respect to interest rates, ζ. We consider several of the strategies to calibrate ζ that have been proposed in the literature and use an indirect-inference step to determine the technology shock parameters.

Neutral Technology Shock Parameters. In the case of constant capital utilization, we constructed a sequence for capital, and in turn the neutral technology shock, based on investment data by iterating the capital accumulation equation (3) forward with $\delta(u_t) = .013$. In the case of variable capital utilization, the capital depreciation rate is time-varying and the use of the perpetual inventory method requires a utilization series as well as knowledge of the function $\delta(u_t)$. Since capital utilization is difficult to measure without error, we proceed with an indirect inference approach. We will use actual data to construct the same series of measured total factor productivity (TFP) that we used in Section 3.2. However, for $\zeta > 0$ this measured TFP cannot be equated with the neutral technology shock because it ignores the effect of time-varying utilization and, hence, it contains an incorrect measure of capital that does not account for utilization-dependent depreciation rates.

Suppose a parameter value for ζ has been selected. Let $\theta_{(-a)} = [\alpha, \beta, \delta_0, \nu, \zeta, \psi_{1,\nu}, \psi_{2,\nu}, \sigma_\nu]'$ denote the DSGE model parameter not associated with the neutral technology process and define $\theta_{(a)} = [\psi_{1,a}, \psi_{2,a}, \sigma_a]'$.[20] We proceed by simulating data from the DSGE model for various choices of $\theta_{(a)}$, compute measured TFP from the simulated data, and estimate the autoregressive coefficients $\psi^m_{1,a}$, $\psi^m_{2,a}$, and σ^m_a for the simulation-based measured TFP series. We then choose the $\theta_{(a)}$ that minimizes a discrepancy measure between the estimated parameters of measured TFP based on the actual and simulated data. As actual parameter estimates (target) we use the posterior means reported in Table 1. Throughout this process, the parameters $\theta_{(-a)}$ are held fixed. Further details are provided in the Online Appendix.

Direct Estimates of ζ. Basu and Kimball (1997) provide direct estimates of $1/\zeta$ using observable proxies for the latent utilization of factor inputs. These proxies are relative factor prices as well as material and energy inputs, and their relationship to utilization has been derived from the firms' cost minimization problem under some assumptions on the form of the production function. Their benchmark point estimates imply a value of ζ of about 1. However, their confidence intervals, without imposing a non-negativity constraint, range from about -0.2 to 2 for $1/\zeta$, which in turn is consistent with large values of ζ. Baxter and Farr (2001) and Mandelman et al. (2011) set the elasticity to $\zeta = 1$, essentially using Basu and Kimball (1997)'s point estimate. Christiano et al. (2005) use a direct, albeit potentially noisy, measure of capital utilization as one of the observables in a structural VAR that is used to generate a set of impulse responses, including that of utilization, to a monetary policy shock. The parameter ζ (among others) is then estimated by matching their model's impulse responses to the ones obtained from the structural VAR. Their estimation procedures insisted on a huge value of the elasticity that attempted to break the feasibility of solving the model. As a compromise, they set $\zeta = 100$.

Turning ζ into a Steady-State-Related Parameter. Greenwood et al. (1988) specify a power functional form for depreciation with only one parameter as a function of capacity utilization,

[20] Since we are interested only in the business cycle properties of the model, the parameters that determine the level of the series, $\ln A_0$ and $\ln V_0$, and the growth rate parameters γ_a and γ_ν are omitted.

and then they set the elasticity to get a steady-state depreciation rate of 10%.[21] The implied value of the elasticity is $\zeta = 2.38$. Similarly, in a model with productivity shocks, endogenous capital utilization, and labor hoarding, Burnside and Eichenbaum (1996) choose $\zeta = 1.85$ to match a steady-state depreciation target given steady-state capital-output ratio, discount rate, and output growth targets. It yields a value $\zeta = 1.85$. Greenwood et al. (2000) have a more complicated model with equipment and structures, and its technological structure is such that to calibrate it, seven parameters are controlled by seven targets. All these targets are based on long-run U.S. features that the balanced growth path of the economy is set to replicate. Their value is $\zeta = 1.69$.

Relative Importance of Technology Shocks. To examine the relevance of the sources of identification of ζ, we consider the following set of values: $\{0, 0.2, 1, 2, 5\}$. The value of 0 corresponds to the benchmark case of constant utilization, and the remaining values are chosen to cover the estimates reviewed above. Our approach resembles that of King and Rebelo (1999), who recognized how little is known about the utilization elasticity and conducted a large sensitivity analysis for values of ζ between 0 to 10. The results for a Frisch labor supply elasticity[22] of $\nu = 0.72$ are summarized in Table 7. The top rows of the table document that the presence of variable capital utilization does not really affect the estimated persistence of the neutral technology shock, captured by $\psi_{1,a}$ and $\psi_{2,a}$, for the range of ζ's considered. However, the estimate of the innovation standard deviation σ_a monotonically declines with ζ. The resulting variation in the contribution of neutral technology shocks to the variance of hours is not very large, ranging from 2% to 4%, and is non-monotonic in ζ.

The measurement of the investment-specific technology shock is not affected by the variable capital utilization. Since the mechanism of capacity utilization tends to amplify hours movements, the variance ratio for hours fluctuations caused by the V shock increases from 6% ($\zeta = 0$) to 21% ($\zeta = 5$) depending on the choice of the parameter, a choice made at best on thin grounds. Thus, as in the benchmark model with constant capital utilization, the choice of identification scheme for a key parameter yields a lot of variation in the quantitative finding.

[21] This is conditional on a discount rate of 0.96.

[22] For each value of ζ, the analysis of the importance of the identification of the Frisch elasticity of labor can be easily replicated. Clearly, Section 3.3 already showed the answers for $\zeta = 0$. For instance, for $\zeta = 1$ we obtain that the variance ratio for hours ranges between 0.01 for $\nu = 0.2$ to 0.54 for $\nu = 2$.

5.2. Bayesian Estimation

In the likelihood-based DSGE model estimation, the utilization elasticity ζ is implicitly identified from the autocovariances of the observables. Variable capital utilization could either be treated as a latent variable, meaning that information about ζ is extracted, say, from the joint dynamics of productivity, hours, and the relative price of investment, or a measure of utilization could be added to the set of observations. Including a utilization series as an observable has the advantage that its cross-correlation with the other time series can provide valuable identifying information for ζ. The disadvantage is that this information is potentially contaminated by measurement errors. We subsequently document the range of estimates that can arise from different ways of treating capital utilization.

Latent Variable Approach. When treating utilization as a latent variable, we only need to include a prior distribution for ζ, which we choose as $\zeta \sim \mathcal{G}(2, 1)$. A 90% credible interval covers all the values considered in the calibration except $\zeta = 5$.

Treating Utilization as Observable. Following the existing literature, two proxies for variable capital utilization are considered: capacity utilization data from industrial production, henceforth *TCU*, and a survey of electric power use, denoted by *E*. The *TCU* series starts in 1967:Q1, whereas the *E* series is available from 1972:Q1 to 2005:Q4. Our estimation sample is adjusted accordingly. *TCU* is divided by its sample average to ensure that the average utilization is normalized to one, and then we take the logarithm. *E* has an upward trend that reflects an increase in electricity intensity of the production process. Consequently, we remove a deterministic trend from the log of electrical power use.

To use the utilization data in the estimation, a measurement equation needs to be specified:

$$u_t^m = \hat{u}_t + \omega_t, \quad \omega_t = \rho_u \omega_{t-1} + \sigma_u \eta_t. \tag{10}$$

Here, u_t^m is the observed utilization and $\hat{u}_t$ is the utilization in the DSGE model in terms of log deviations from steady state. The AR(1) process ω_t captures a measurement error. We consider the following prior distribution for the parameters of the measurement error process:

$$\rho_u \sim B(0.5, 0.2), \quad \sigma_u \sim IG(0.01, 4).$$

At the prior mean, the measurement error variance is less than 1% of the variance of the observed utilization series.

Posteriors and the Relative Importance of Technology Shocks. The estimation results are presented in Table 8. We estimate the $\{A, V, B\}$ specification of the DSGE model on the $\{Y/H, H, P\}$ sample, and the $\{A, V, B, \omega\}$ specification on the $\{Y/H, H, P, TCU\}$ and $\{Y/H, H, P, E\}$ samples. The latter specification includes the capital utilization measurement error process ω_t. The estimates of ζ are very sensitive to the identifying information contained in the three samples and vary considerably. The estimated utilization elasticity is 0.24 if utilization is treated as a latent variable. Since ζ affects the shape of the impulse responses and the autocovariance functions, it is identifiable in the absence of utilization data. If utilization data are included into the set of observables, the estimate of ζ increases to 0.75 (E) and 1.70 (TCU), respectively. However, the estimated autocorrelation of the measurement error process is around 0.95, and the estimate of σ_u is around 0.014. This implies that more than 90% of the variability in the utilization series is attributed to measurement errors, suggesting a mismatch between model-implied and observed utilization dynamics.

The estimated first-order partial autocorrelation of the neutral technology process drops from 0.98 to 0.96 if utilization data are included, and the standard deviation of the innovation drops from .0068 to .0057. Inference with respect to the labor supply elasticity ν is very similar across samples. The estimates are slightly lower than those obtained from the $\{Y/H, H, P\}$ sample reported in Table 4. The discussion in Section 4.3, in particular Equation (9), implies that under variable capital utilization, the labor supply elasticity remains identifiable based on the relative response of labor productivity and hours to a technology shock. The identifying information is contained in the relative response of wages and hours to technology shocks and thus encoded in the likelihood function even if utilization is excluded from the set of observables.

The estimated contribution of technology shocks to fluctuations of hours worked and output is less sensitive to the choice of data set than the estimation of the utilization elasticity. Overall, the variance ratios remain in the range of 6% to 11% for hours worked and 24% to 28% for output. Unlike in the calibration, identification of the labor supply elasticity and the utilization elasticity are linked, and the amplification effect of variable capital utilization is essentially offset by a lower estimate of the labor supply elasticity.

5.3. Further Generalizations Considered in the Literature

A recent literature uses an expanded version of the neoclassical growth model, includes various nominal and real frictions as well as several additional shocks, and provides an answer to the same question that we pursue. Justiniano et al. (2010b) estimate one of these expanded models with the same Bayesian techniques used in this paper. They assess the contribution of technology shocks (especially investment-specific shocks) to be up to 60% of hours variation at business cycle frequencies. The reasons for this discrepancy with our findings include the fact that in their environment with fixed wages, a fraction of agents is unable to reoptimize their price and forced to supply whatever number of hours is demanded at the posted price; thus, even with small Frisch elasticity, hours tend to move a lot in response to technology shocks. Another source of discrepancy, at least compared with our estimation with the $\{Y/H, H, P\}$ and $\{Y/H, H, P, X\}$ data sets, is the fact that they treat the investment shock as a latent process, which turns out to be much more volatile than the relative price of investment. Justiniano et al. (2010a) include the relative price of investment as an observable but allow for an additional (unobserved) shock to the marginal efficiency of installed investment. They find that this shock plays a big role in accounting for hours variation relative to the observed shocks to the relative price of investment. Liu et al. (2009) impute most of the role in shaping fluctuations (especially when focusing on the Great Moderation) to the role of neutral technological shocks, capital depreciation shocks, and wage mark-up shocks, while they argue that investment-specific technology shocks played a small role.

6. Lessons for Practitioners

While our methodological points were presented in the context of a specific application, there are several lessons learned for other models and applications.

Identification First, Methods Second. The first step of the analysis should be to consider various plausible ways of identifying those parameters that are highly influential in shaping the quantitative findings. The second step is to incorporate these identification schemes into the quantitative analysis, be it a calibration or an econometric estimation of the DSGE model. Unfortunately, much of the existing literature has the order reversed: researchers make a choice as to whether to calibrate or estimate the model, and then rely on the identification approaches that are typically associated with these methodologies. We deliberately followed this reverse order in our presentation to highlight three identification schemes for an aggregate labor supply elasticity that are typically associated with a calibration and to provide some new insights into the identification schemes that are hardwired into different likelihood functions. However, as a recommendation for practitioners, we suggest disassociating identification schemes from quantitative methodologies.

Watch out for the 800-Pound Gorilla. In a perfect world, different sources of identifying information would yield mutually consistent conclusions about key model parameters after the precision of the information has been properly accounted for. Unfortunately, that is not the case if one works with fairly stylized and to some extent misspecified models, as we do in quantitative macroeconomics.[23] The reality is that different identification approaches often yield conflicting quantitative findings. Much of the literature on estimated DSGE models concentrates on characterizing uncertainty conditional on a particular identification scheme but loses track of the often more important sensitivity of results to the initial choice of identification source.

To Calibrate or Estimate? The authors disagree on thie question of whether to calibrate or estimate. The main difference between calibration and estimation is that the latter assigns statistical weights to different sources of information. One example is the optimal weight matrix for overidentifying moment conditions in a Generalized Methods of Moments (GMM) setting, which tries to attach Bayesian or frequentist measures of uncertainty to the quantitative results.

The authors do agree that the various identification sources that we highlighted in the context of our application and that arise in other applications can be incorporated in either a calibration or an estimation approach. For instance, a calibration could use the impulse response information from Section 4.3 to parameterize the labor supply elasticity. The identification of the neutral technology shock parameters in Section 5.1 mimics the indirect inference approach in econometrics developed by Gourieroux et al. (1993) and Smith

[23] Table 6 also contains log marginal likelihood differences for a VAR(4) with Minnesota prior. For all three data sets, the VAR attains a substantially better fit than the DSGE model specifications, which is evidence for DSGE model misspecification. Some of the theory-implied cross-equation restrictions are at odds with the data. The same is true for more sophisticated DSGE models as documented in Del Negro et al. (2007).

(1993). Estimation can use information from micro-level data, impose functional form restrictions that turn endogenous propagation parameters into steady-state-related parameters, or extract information from long-run averages of interest rates, capital-investment ratios, or consumption-investment ratios. This can be implemented in part by using prior distribution in a Bayesian framework or through setting up moment conditions in a frequentist GMM or minimum-distance framework.

Aggregating Results. Econometric approaches, in particular Bayesian approaches, tend to weight quantitative predictions from different model specifications by the relative fit that these specifications attain. In our application, we used Bayesian model averaging to combine results from different DSGE model specifications estimated based on the same data set. This proved to be a convenient tool but did not allow us to aggregate results obtained from different data sets. Across data sets, the variation in hours explained by technology shocks ranges from 1% to 5% and the variation in output from 21% to 25%.

For the calibration analysis, we tabulated our findings conditional on various choices of the labor supply elasticity, enabling the reader to apply her or his own weighting scheme for the plausibility of the various identification approaches. Our own weighting scheme takes the following form: we have some doubts about the two extreme values of the labor supply elasticity, which are based on middle-aged full-time working white men, the most irrelevant group from a business cycle point of view or in unnecessary, and probably misguided restrictions from balanced growth paths. After all, it is hard to see what insights long-term trends generate for business cycle issues. The use of micro-based estimates that take into account both the work of men and women and the intensive and extensive margins as in Heathcote et al. (2007, 2008) deliver arguably the most plausible estimates of ν and imply that between 3% and 9% of hours fluctuations are explained by technology shocks. These estimates happen to be consistent with those from the likelihood-based analysis.

In the environment with capital utilization, the calibration approach for a unit value of ζ (the elasticity of utilization with respect to the interest rate) and for our favorite labor elasticity yields an estimate of the variance of hours accounted for by technology shocks of 0.15. The estimation approach yields estimates ranging from 0.06 to 0.11. Only when the utilization elasticity is set to 5 does the calibration yield very different answers.

7. Conclusion

The main contribution of our paper is to compare how key model parameters are identified in a standard calibration versus a likelihood-based estimation. We make the case that quantitative macroeconomists will benefit from thinking about particular identification sources independently of the quantitative approaches that are used to exploit them. In the context of a specific application, we shed some light on how the likelihood function extracts information about important model parameters and thus contribute to the growing econometric literature on DSGE model identification. Moreover, we carefully examine the sensitivity of likelihood-based estimates to the inclusion of additional shocks that complete the probabilistic structure of the DSGE model, and we use Bayesian model averaging to aggregate the quantitative implications of the various model specifications. The analysis conducted in this paper has convinced us that, regardless of our preferences for quantitative methodologies, we should place more emphasis on searching for reliable sources of identification of key parameters and making these sources transparent to our audience.

Altonji, J. G., 1986. Intertemporal substitution in labor supply: evidence from micro data. Journal of Political Economy 94 (3), 176–215.

An, S., Schorfheide, F., 2007. Bayesian analysis of DSGE models. Econometric Reviews 26 (2-4), 113–172.

Basu, S., Kimball, M. S., 1997. Cyclical productivity with unobserved input variation. NBER Working Paper 5915.

Baxter, M., Farr, D. D., 2001. Variable factor utilization and international business cycles. NBER Working Paper 8392.

Browning, M., Deaton, A., Irish, M., 1985. A profitable approach to labor supply and commodity demands over the life-cycle. Econometrica 53 (3), 503–543.

Burnside, C., Eichenbaum, M., 1996. Factor-hoarding and the propagation of business-cycle shocks. American Economic Review 86 (5), 1154–1174.

Canova, F., Lopez-Salido, D., Michelacci, C., 2010. The effects of technology shocks on hours and output: a robustness analysis. Journal of Applied Econometrics 25 (5), 755–773.

Canova, F., Sala, L., 2009. Back to square one: identification issues in DSGE models. Journal of Monetary Economics 56 (4), 431–449.

Chang, Y., Gomes, J. F., Schorfheide, F., 2002. Learning-by-doing as a propagation mechanism. American Economic Review 92 (5), 1498–1520.

Chang, Y., Kim, S.-B., 2006. From individual to aggregate labor supply: a quantitative analysis based on a heterogeneous agent macroeconomy. International Economic Review 47 (1), 1–27.

Chari, V. V., Kehoe, P. J., McGrattan, E. R., 2007. Business cycle accounting. Econometrica 75 (3), 781–836.

Christiano, L. J., Eichenbaum, M., Evans, C. L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113 (1), 1–45.

Cooley, T. F., Prescott, E. C., 1995. Economic growth and business cycles. In: Cooley, T. F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, NJ, pp. 1–38.

Cummins, J. G., Violante, G. L., 2002. Investment-specific technical change in the United States (1947-2000): measurement and macroeconomic consequences. Review of Economic Dynamics 5 (2), 243–284.

Del Negro, M., Schorfheide, F., 2008. Forming priors for DSGE models (and how it affects the assessment of nominal rigidities). Journal of Monetary Economics 55 (7), 1191–1208.

Del Negro, M., Schorfheide, F., Smets, F., Wouters, R., 2007. On the fit and forecasting performance of New Keynesian models. Journal of Business and Economic Statistics 25 (2), 123–162.

Domeij, D., Floden, M., 2006. The labor-supply elasticity and borrowing constraints: why estimates are biased. Review of Economic Dynamics 9 (2), 242–262.

Fisher, J. D. M., 2006. The dynamic effects of neutral and investment-specific technology shocks. Journal of Political Economy 114 (3), 413–451.

Galí, J., Rabanal, P., 2005. Technology shocks and aggregate fluctuations: how well does the real business cycle model fit postwar U.S. data? In: Gertler, M., Rogoff, K. (Eds.), NBER Macroeconomics Annual 2004. Vol. 19. MIT Press, Cambridge, MA, pp. 225–288.

Gordon, R. J., 1990. The Measurement of Durable Goods Prices. National Bureau of Economic Research Monograph Series, University of Chicago Press, Chicago.

Gourieroux, C., Monfort, A., Renault, E., 1993. Indirect inference. Journal of Applied Econometrics 8 (S1), S85–S118.

Greenwood, J., Hercowitz, Z., Huffman, G. W., 1988. Investment, capacity utilization, and the real business cycle. American Economic Review 78 (3), 402–418.

Greenwood, J., Hercowitz, Z., Krusell, P., 1997. Long-run implications of investment-specific technological change. American Economic Review 87 (3), 342–362.

Greenwood, J., Hercowitz, Z., Krusell, P., 2000. The role of investment-specific technological change in the business cycle. European Economic Review 44 (1), 91–115.

Guerron-Quintana, P. A., 2010. What you match does matter: the effects of data on DSGE estimation. Journal of Applied Econometrics 25 (5), 774–804.

Hall, R. E., 1997. Macroeconomic fluctuations and the allocation of time. Journal of Labor Economics 15 (1), S223–S520.

Hansen, G., 1985. Indivisible labor and the business cycle. Journal of Monetary Economics 16 (3), 309–327.

Hansen, L. P., Heckman, J. J., 1996. The empirical foundations of calibration. Journal of Economic Perspectives 10 (1), 87–104.

Heathcote, J., Storesletten, K., Violante, G., 2007. Consumption and labor supply with partial insurance: an analytical framework. University of Oslo, Frisch Centre (Oslo) Working Paper.

Heathcote, J., Storesletten, K., Violante, G., 2008. The macroeconomic implications of rising wage inequality in the united states. University of Oslo, Frisch Centre (Oslo) Working Paper.

Imai, S., Keane, M. P., 2004. Intertemporal labor supply and human capital accumulation. International Economic Review 45 (2), 601–641.

Iskrev, N., 2010. Local identification in DSGE models. Journal of Monetary Economics 57 (2), 189–202.

Justiniano, A., Primiceri, G. E., Tambalotti, A., 2010a. Investment shocks and business cycles. Journal of Monetary Economics 57 (2), 132–145.

Justiniano, A., Primiceri, G. E., Tambalotti, A., 2010b. Investment shocks and the relative price of investment. Review of Economic Dynamics 14 (1), 102–121.

Keane, M. P., 2010. Labor supply and taxes: a survey. Working Paper 160, School of Finance and Economics, University of Technology Sydney.

King, R. G., Plosser, C. I., Rebelo, S. T., 1988. Production, growth and business cycles I: the basic neoclassical model. Journal of Monetary Economics 21 (2-3), 195–232.

King, R. G., Rebelo, S. T., 1999. Resuscitating real business cycles. In: Taylor, J. B., Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier, Amsterdam, pp. 927–1007.

Komunjer, I., Ng, S., 2009. Dynamic identification of DSGE models. Manuscript, Columbia University and UCSD.

Kydland, F. E., 1984. Labor-force heterogeneity and the business cycle. Carnegie-Rochester Conference Series on Public Policy 21 (1), 173–209.

Kydland, F. E., Prescott, E. C., 1982. Time to build and aggregate fluctuations. Econometrica 50 (6), 1345–1370.

Kydland, F. E., Prescott, E. C., 1988. Cyclical movements of the labor input and its real wage. Federal Reserve Bank of Minneapolis Working Paper 413.

Kydland, F. E., Prescott, E. C., 1991. Hours and employment variation in business cycle theory. Economic Theory 1 (1), 63–81.

Kydland, F. E., Prescott, E. C., 1996. The computational experiment: an econometric tool. Journal of Economic Perspectives 10 (1), 69–85.

Liu, Z., Waggoner, D. F., Zha, T., 2009. Sources of the Great Moderation: shocks, frictions, or monetary policy? Tech. rep., Federal Reserve Bank of Atlanta Working Paper 2009-03. URL http://ideas.repec.org/p/fip/fedawp/2009-03.html

Low, H. W., 2005. Self-insurance in a life-cycle model of labour supply and savings. Review of Economic Dynamics 8 (4), 945–975.

MaCurdy, T. E., 1981. An empirical model of labor supply in a life-cycle setting. Journal of Political Economy 89 (6), 1059–1085.

Mandelman, F. S., Rabanal, P., Rubio-Ramírez, J. F., Vilán, D., 2011. Investment-specific technology shocks and international business cycles: an empirical assessment. Review of Economic Dynamics 14 (1), 136–155.

Pencavel, J., 1986. Labor supply of men: a survey. In: Ashenfelter, O., Layard, R. (Eds.), Handbook of Labor Economics, vol. 1. Elsevier, Amsterdam, pp. 3–102.

Ríos-Rull, J.-V., 1993. Working in the market, working at home, and the acquisition of skills: a general-equilibrium approach. American Economic Review 83 (4), 893–907.

Ríos-Rull, J.-V., Santaeulàlia-Llopis, R., 2010. Redistributive shocks and productivity shocks. Journal of Monetary Economics 57 (8), 931–948.

Rogerson, R., 1988. Indivisible labor, lotteries and equilibrium. Journal of Monetary Economics 21 (1), 3–16.

Rogerson, R., Wallenius, J., 2010. Fixed costs, retirement and the elasticity of labor supply. Manuscript, Arizona State University.

Shimer, R., 2009. Convergence in macroeconomics: the labor wedge. American Economic Journal: Macroeconomics 1 (1), 280–297.

Sims, C., 1996. Macroeconomics and methodology. Journal of Economic Perspectives 10 (1), 105–120.

Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro area. Journal of the European Economic Association 1 (5), 1123–1175.

Smets, F., Wouters, R., 2007. Shocks and frictions in U.S. business cycles: a Bayesian DSGE approach. American Economic Review 97 (3), 586–606.

Smith, A. A., 1993. Estimating nonlinear time-series models using simulated vector autoregressions. Journal of Applied Econometrics 8 (S1), S63–S84.

Trabandt, M., Uhlig, H., 2009. How far are we from the slippery slope? The Laffer curve revisited. NBER Working Paper 15343.

Table 1: Priors and Posteriors for Technology Shock Parameters

		Prior Distribution			Deterministic Trend		Stochastic Trend	
Name	Domain	Density	Para (1)	Para (2)	Mean	90 % Intv.	Mean	90 % Intv.
γ_a	$I\!R$	Normal	0.00	0.10	-.001	[-.002, .000]	.000	[-.001, .001]
$\psi_{1,a}$	$[0,1)$	Beta	0.95	0.02	0.97	[0.96, 0.99]	1.00	
$\psi_{2,a}$	$(-1,1)$	Uniform	-1.0	1.00	-0.03	[-0.15, 0.09]	-0.06	[-0.19, 0.05]
σ_a	$I\!R^+$	InvGamma	0.01	4.00	.007	[.006, .008]	.007	[.006, .008]
γ_v	$I\!R$	Normal	0.00	0.10	.008	[.007, .008]	.007	[.005, .009]
$\psi_{1,v}$	$[0,1)$	Beta	0.95	0.02	0.99	[0.99, 1.00]	1.00	
$\psi_{2,v}$	$(-1,1)$	Uniform	-1.0	1.00	-0.76	[-0.84, -0.69]	-0.81	[-0.90, -0.73]
σ_v	$I\!R^+$	InvGamma	0.01	4.00	.003	[.003, .004]	.003	[.003, .004]
$\ln A_0$	$I\!R$	Normal	0.00	100	4.84	[4.74, 4.95]	-2.66	[-97.4, 76.5]
$\ln V_0$	$I\!R$	Normal	0.00	100	-0.14	[-0.24, -0.06]	-0.85	[-79.9, 86.1]

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last four columns contain posterior means and 90% credible intervals. To estimate the stochastic trend version of the model, we set $\psi_{1,a} = \psi_{1,v} = 1$.

Table 2: CALIBRATION

		(Old) Micro Studies of Males, Intensive Margin $\nu = 0.2$		(New) Micro Studies of Males, Intensive Margin $\nu = 0.72$		Nonconvexities Households, All Margins $\nu = 1.0$		Balanced Growth Cobb-Douglas Utility $\nu = 2.0$	
		Deterministic Trend							
		$\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\gamma_a = -0.001$, $\psi_{1,a} = 0.97$, $\psi_{2,a} = -0.03$, $\sigma_a = 0.007$, $\gamma_v = 0.008$, $\psi_{1,v} = 0.99$, $\psi_{2,v} = -0.76$, $\sigma_v = 0.003$							
Series	Shock	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
Hours	A	.004	(.001)	0.03	(.005)	0.05	(.008)	0.011	(.017)
	V	.006	(.002)	0.06	(.014)	0.09	(.021)	0.21	(.050)
	A+V	**0.01**	(.002)	**0.09**	(.018)	**0.14**	(.028)	**0.32**	(.067)
Output	A	0.20	(.017)	0.26	(.021)	0.28	(.023)	0.35	(.026)
	V	.001	(.000)	.004	(.001)	.007	(.001)	0.02	(.002
	A+V	**0.20**	(.017)	**0.26**	(.022)	**0.29**	(.024)	**0.37**	(.030)
		Stochastic Trend							
		$\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\gamma_a = 0$, $\psi_{1,a} = 1$, $\psi_{2,a} = -0.06$, $\sigma_a = 0.007$, $\gamma_v = 0.007$, $\psi_{1,v} = 1$, $\psi_{2,v} = -0.81$, $\sigma_v = 0.003$							
Series	Shock	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
Hours	A	.001	(.000)	0.01	(.002)	0.02	(.003)	0.04	(.006)
	V	.006	(.001)	0.06	(.009)	0.10	(.015)	0.25	(.036)
	A+V	**.008**	(.015)	**0.07**	(.011)	**0.11**	(.018)	**0.29**	(.042)
Output	A	0.18	(.015)	0.21	(.018)	0.22	(.019)	0.25	(.021)
	V	.002	(.000)	.018	(.001)	0.03	(.003)	0.08	(.007)
	A+V	**0.19**	(.015)	**0.23**	(.019)	**0.25**	(.022)	**0.33**	(.027)

Note: We report variance ratios (simulated/actual) for HP-filtered series.

Table 3: Prior Distribution for DSGE Model Parameters

Name	Domain	Density	Para (1)	Para (2)
α	$[0, 1)$	Beta	0.34	0.02
β		fixed	0.99	
δ_0		fixed	.013	
ν	$I\!R^+$	Gamma	2.00	1.00
ρ_b	$[0, 1)$	Beta	0.95	0.02
σ_b	$I\!R^+$	InvGamma	.017 or .012	4.00
g^*		fixed	1.00 or 1.20	
ρ_g	$[0, 1)$	Beta	0.95	0.02
σ_g	$I\!R^+$	InvGamma	.010 or .007	4.00
$\ln H_*$	$I\!R$	Normal	0.00	10.0
$\ln Y_0$	$I\!R$	Normal	0.00	100

Notes: Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; and s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. To estimate the stochastic growth version of the model, we set $\psi_{1,a} = \psi_{1,v} = 1$. The $\{A, V, B\}$ (benchmark) specification is estimated with $Para(1) = .017$ σ_b prior. The $\{A, V, G\}$ specification is estimated with $Para(1) = .010$ σ_g prior. The $\{A, V, B, G\}$ specification is estimated with $Para(1) = .012$ σ_b prior and $Para(1) = .007$ σ_g prior.

Table 4: Labor Supply Elasticity Estimates

Unit Roots Imposed	Shocks	Data Set Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
		Mean	90% Intv.	Mean	90% Intv.	Mean	90% Intv.
No	A, V, B	0.85	[0.34, 1.33]	0.82	[0.44, 1.17]		
	A, V, G	0.63	[0.35, 0.89]	1.56	[0.93, 2.17]		
	A, V, B, G	0.70	[0.33, 1.05]	0.77	[0.38, 1.14]	0.17	[0.05, 0.28]
Yes	A, V, B	0.30	[0.06, 0.53]	0.60	[0.30, 0.88]		
	A, V, G	0.42	[0.17, 0.64]	1.83	[1.05, 2.60]		
	A, V, B, G	0.35	[0.07, 0.63]	0.96	[0.33, 1.54]	0.12	[0.03, 0.22]

Table 5: Importance of Technology Shocks

Unit Roots Imposed	Shocks	Series	Data Set Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
			Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
No	A, V, B	Hours	0.11	(.066)	**0.05**	(.025)		
		Output	0.26	(.040)	**0.26**	(.038)		
	A, V, G	Hours	**0.08**	(.046)	0.22	(.084)		
		Output	**0.25**	(.037)	0.33	(.047)		
	A, V, B, G	Hours	0.10	(.060)	0.07	(.038)	**0.01**	(.008)
		Output	0.26	(.040)	0.27	(.043)	**0.22**	(.031)
Yes	A, V, B	Hours	0.01	(.009)	0.03	(.015)		
		Output	0.20	(.028)	0.23	(.033)		
	A, V, G	Hours	**0.02**	(.010)	0.20	(.063)		
		Output	**0.21**	(.029)	0.32	(.044)		
	A, V, B, G	Hours	**0.02**	(.014)	**0.05**	(.030)	.002	(.002)
		Output	**0.21**	(.030)	**0.25**	(.040)	0.19	(.026)
Weighted		Hours	**0.02**		**0.05**		**0.01**	
		Output	**0.21**		**0.25**		**0.22**	

Notes: Variance ratios are in bold for model specifications that attain a posterior model probability of 3% or more. The last two rows (Weighted) contain weighted averages based on the marginal likelihoods in Table 6.

Table 6: Log Marginal Likelihood Differentials and Posterior Probabilities

Unit Roots Imposed	Shocks	Data Set: Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
		$\ln p(Y)$	(Prob.)	$\ln p(Y)$	(Prob.)	$\ln p(Y)$	(Prob.)
No	A, V, B	-11.7	(.000)	-3.32	(.035)		
	A, V, G	-3.38	(.032)	-12.6	(.000)		
	A, V, B, G	-6.01	(.002)	-5.63	(.003)	0.00	(.992)
Yes	A, V, B	-8.67	(.000)	-7.33	(.001)		
	A, V, G	0.00	(.932)	-15.01	(.000)		
	A, V, B, G	-3.32	(.034)	0.00	(.961)	-4.82	(.008)
VAR(4), Minnesota Prior		54.72		28.97		57.30	

Notes: For each data set, the log marginal likelihood differences are computed relative to the DSGE model specification with the highest marginal likelihood. The log marginal likelihoods for these specifications are 2278.12, 1951.07, and 2820.11, respectively. The calculation of posterior probabilities is based on equal prior probabilities and excludes the VAR.

Table 7: CALIBRATION OF VARIABLE CAPITAL UTILIZATION MODEL ($\nu = 0.72$)

	$\zeta = 0$		$\zeta = 0.2$		$\zeta = 1$		$\zeta = 2$		$\zeta = 5$	
Neutral Technology Shock Parameters										
$\psi_{a,1}$	0.974		0.974		0.985		0.9846		0.985	
$\psi_{a,2}$	-0.027		-0.027		-0.004		0.0367		0.094	
σ_a	.0070		.0067		.0053		0.0048		.0043	
Decomposition of Hours										
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.03	(.005)	0.04	(.006)	0.02	(.004)	0.02	(.003)	0.02	(.003)
V	0.06	(.014)	0.08	(.020)	0.12	(.030)	0.16	(.039)	0.21	(.055)
A,V	**0.09**	(.018)	**0.12**	(.025)	**0.15**	(.033)	**0.18**	(.042)	**0.23**	(.057)
Decomposition of Output										
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.26	(.021)	0.28	(.023)	0.23	(.019)	0.23	(.020)	0.24	(.021)
V	.004	(.001)	.007	(.001)	0.02	(.003)	0.03	(.005)	0.05	(.009)
A,V	**0.26**	(.022)	**0.29**	(.024)	**0.25**	(.021)	**0.27**	(.022)	**0.29**	(.025)

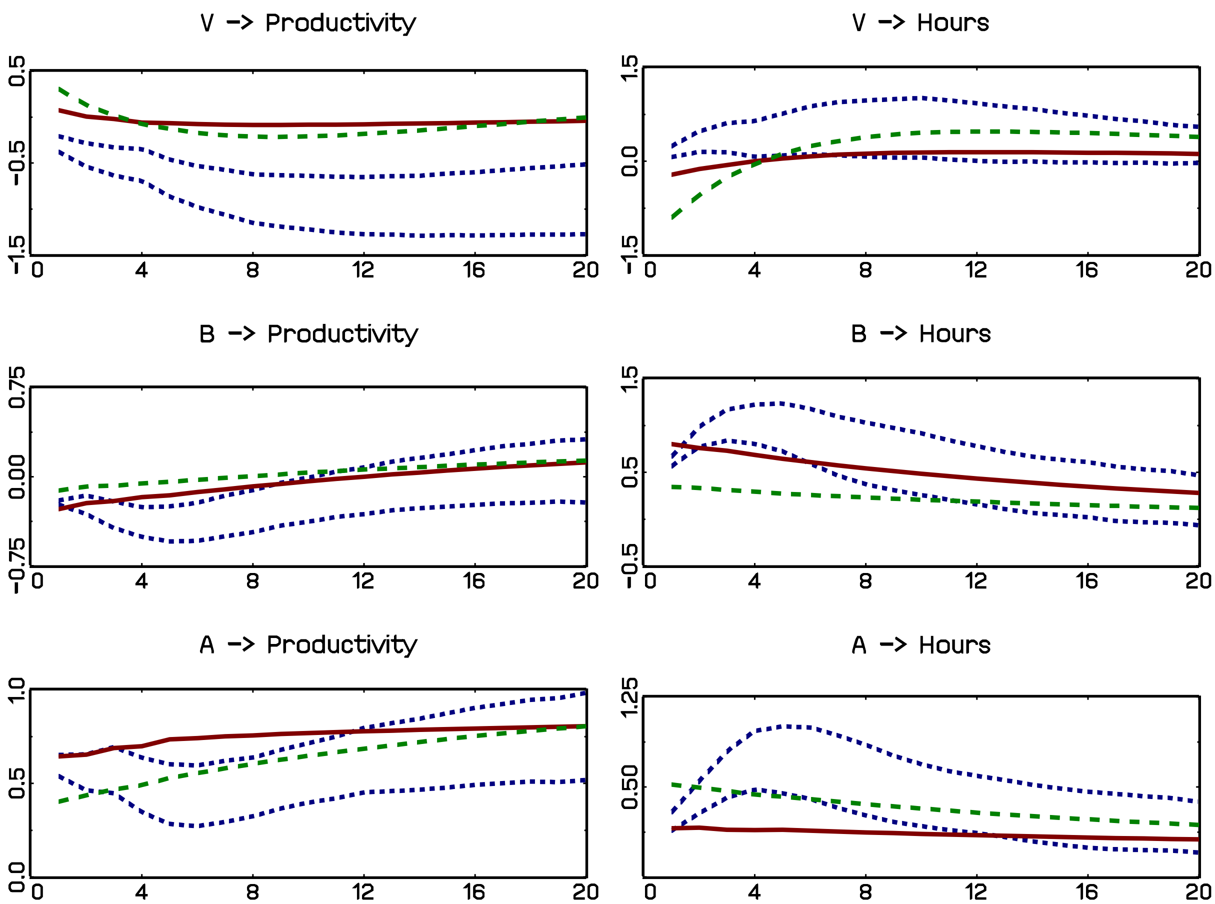
Notes: ζ is the elasticity of capital utilization with respect to the rental rate. The parameter values for $\theta_{(-a)}$ are the same as in Table 2: $\alpha = 0.34$, $\beta = 0.99$, $\delta_0 = 0.013$, $\psi_{1,v} = 0.99$, $\psi_{2,v} = -0.76$, $\sigma_v = 0.003$. The first column ($\zeta = 0$) corresponds to the case of constant utilization and reproduces the variance ratios from Table 2. Since for $\zeta = 0$ measured TFP corresponds to the neutral technology shock, the values for $\theta_{(a)}$ correspond to the posterior means reported in Table 1.

Table 8: Estimation of Variable Capital Utilization Model

	Data Set					
	Y/H, H, P		Y/H, H, P, TCU		Y/H, H, P, E	
	Selected Parameter Estimates					
	Mean	90% Intv.	Mean	90% Intv.	Mean	90% Intv.
ν	0.457	[0.158, 0.737]	0.346	[0.105, 0.570]	0.459	[0.160, 0.753]
ζ	0.238	[0.047, 0.426]	1.703	[0.969, 2.431]	0.746	[0.310, 1.168]
$\psi_{a,1}$	0.984	[0.975, 0.994]	0.981	[0.965, 0.996]	0.964	[0.941, 0.989]
$\psi_{a,2}$	-0.110	[-0.247, 0.016]	-0.130	[-0.274, 0.011]	-0.168	[-0.322, 0.012]
σ_a	.0068	[.0062, .0074]	.0057	[.0052, .0062]	.0057	[.0051, .0062]
	Decomposition of Hours					
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.02	(.015)	0.01	(.094)	0.02	(.022)
V	0.04	(.032)	0.07	(.076)	0.10	(.099)
A,V	0.06	(.042)	0.09	(.082)	0.12	(.110)
	Decomposition of Output					
	Mean	(StdD)	Mean	(StdD)	Mean	(StdD)
A	0.24	(.038)	0.26	(.045)	0.24	(.046)
V	.005	(.003)	0.02	(.012)	0.02	(.011)
A,V	0.24	(.039)	0.28	(.051)	0.25	(.052)

Notes: The estimation based on the $\{Y/H, H, P\}$ data set treats capital utilization as a latent variable. In the other two estimations, utilization is treated as observable (with measurement error). The data set includes either capacity utilization data from industrial production (TCU) or electricity usage survey data (E).

Figure 1: VAR Responses, Actual versus Simulated Data



Note: The figure depicts 90% credible bands for a VAR(4) (dotted, blue) estimated based on actual data and posterior mean responses for VAR(4)'s estimated on long samples of DSGE model generated observations with $\nu = 0.2$ (red, solid) and $\nu = 2.0$ (green, dashed).

Online Appendix

Appendix A. Data Construction

Appendix A.1. Raw Data Series

All raw data series retrieved from the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1948:Q1–2006:Q4 were current as of April 19, 2007.

National Income and Product Accounts (NIPA-BEA)

1. Table 1.1.5: Consumption of Durable Goods (CD_t), Change in Inventories (ChInv_t)
2. Table 1.7.5: Gross National Product (GNP_t)
3. Tables 2.3.3 and 2.3.5: Quantity Index (QCONS_t^i) and Nominal (CONS_t^i) Nondurables Consumption (excluding Energy) and Services (excluding Housing)[24]
4. Table 3.9.5: Government Investment in Equipment (GovIEQ_t), Government Investment in Structures (GovIST_t)
5. Table 5.3.5: Private Fixed Investment in Equipment (PrivIEQ_t), Private Fixed Investment in Structures (PrivIST_t)

Fixed Asset Tables (FAT-BEA)

1. Table 5.3.4: Official Price Index for Investment in Equipment (OPIEQ_t)

Bureau of Labor Statistics(BLS)

1. Aggregate Hours Index (H_t), BLS ID PRS85006033
2. Civilian Noninstitutional Population +16 ($Pop16_t$), BLS ID LNU00000000

Cummins and Violante (2002), 1947–2000

1. Annual Quality-Adjusted Price Index for Investment in Equipment ($\text{QAPIEQ}_{year}^{CV}$)
2. Annual Quality-Adjusted Depreciation Rates for Total Capital (δ_{year}^{CV})

[24]Goods i correspond to nondurables consumption in food, clothing and shoes, and others, and services in household operations, transportation, medical care, recreation, and others.

Capital Utilization Data

The data is available from the Statistics & Historical Data page for Principle Economic Indicators – Industrial Production and Capacity Utilization (G.17) at *www.federalreserve.gov/econresdata/releases/statisticsdata.htm*.

1. Electric Power Use: Manufacturing and Mining, Total industry from the survey of industrial electric power conducted by the Board of Governors of the Federal Reserve System. The voluntary survey was discontinued with the publication on December 15, 2005, of data for October 2005 since the response rate dropped significantly during the early 2000s.
2. TCU: capacity utilization: total industry provided by the Board of Governors of the Federal Reserve System. The Federal Reserve Board constructs estimates of capacity utilization for a given industry by dividing an output index by a capacity index. Capacity indexes try to capture the so-called sustainable maximum output, that is, the largest level of output that a plant can achieve given the resources available when operating the plant. Capacity indexes are constructed for 89 detailed industries (71 in manufacturing, 16 in mining, and 2 in utilities), which mostly correspond to industries at the three- and four-digit NAICS level. In the estimation exercise, we use quarterly averages of the monthly series on the percent capacity. The data are available from 1967:1.

Appendix A.2. The Relative Price of Quality-Adjusted Investment

We construct the relative price of quality-adjusted investment, P_t^I, as a Tornquist aggregate of the price index of quality-adjusted equipment investment and the price index of structures investment. We use the price index of consumption, P_t^C, as a proxy for the price of structures investment.[25] Based on P_t^I and P_t^C, we define the relative price of investment goods (using the consumption good as numeraire) as

$$P_t = \frac{P_t^I}{P_t^C}.$$

[25] As is the standard in previous literature, we use the consumption deflator as the price index for investment in structures (see Fisher (2006) and Canova et al. (2010)). This provides internal consistency in the way we compute the quality-adjusted price index for total (equipment + investment) investment —one of the elements of output is investment; hence, the alternative use of an output (instead of a consumption) deflator potentially distorts the very same measure we are trying to compute: an investment deflator.

Its inverse, $V_t = \frac{1}{P_t}$, is investment-specific technical change. We set $V_0 = \frac{1}{P_0} = 1$, that is, we assume real capital is equal to capital in efficiency units in 1947.

Quarterly Quality-Adjusted Price Index for Investment in Equipment, QAPIEQ_t. We use the U.S. 1947-2000 annual series provided by Cummins and Violante (2002) for the price index of equipment investment, $\text{QAPIEQ}^{CV}_{year}$, and impute the quarterly movements of the official FAT-BEA price index of equipment investment, OPIEQ_t, using the Denton method. For the years after 2000, we use the official price index OPIEQ_t, rescaled such that it equates the value in Cummins and Violante (2002) in the year 2000. Thus, we assume that the hedonic methods used to compute the official price index correctly quality-adjust most types of equipment investment after 2000.

Quarterly Quality-Adjusted Price Index for Total Investment, P^I_t. We use a Tornquist price index aggregate that weights growth rates of the price index of investment in equipment and the price index of investment in structures by their nominal shares s^{IEQ}_t and s^{IST}_t. Nominal equipment investment is the sum of private equipment investment (PrivIEQ_t), government equipment investment (GovIEQ_t), changes in inventories (ChInv_t), and consumer durables (CD_t). Nominal structures investment is the sum of private structures investment (PrivIST_t) and government structures investment (GovIST_t). The growth rate of the quarterly quality-adjusted price index for total investment is

$$\lambda(P^I_t) = \left(\frac{s^{IEQ}_t + s^{IEQ}_{t-1}}{2}\right)\lambda(\text{QAPIEQ}_t) \;+\; \left(\frac{s^{IST}_t + s^{IST}_{t-1}}{2}\right)\lambda(P^C_t),$$

where $\lambda(x_t) = (x_t - x_{t-1})/x_t$ and changes in the price index for consumption goods, $\lambda(P^C_t)$, serve as proxy for inflation in the price of structures. The level of quarterly quality-adjusted price index for total investment is recovered recursively,

$$P^I_t = P^I_{t-1}[1 + \lambda(P^I_t)].$$

We use the initial value P^I_0 suggested in Cummins and Violante (2002).

Quarterly Price Index for Consumption, P^C_t. We use a Tornquist price index aggregate that weights growth rates of price indexes for nondurables consumption (food, clothing and shoes, and others) and services (household operations, transportation, medical care, recreation, and others) by their nominal shares. Let $P^{C,i}_t$ be the price index for nondurable consumption/service good i in quarter t computed as the ratio between nominal consumption of good i, CONS_t^i, and the quantity index of good i, QCONS_t^i. Let s_t^i be the corresponding nominal share of good i in period t. Then, the growth rate of the price index for consumption is

$$\lambda(P_t^C) = \sum_i \frac{s_t^i + s_{t-1}^i}{2} \lambda(P_t^{C,i}).$$

The level of the consumption price index is recovered recursively,

$$P_t^C = P_{t-1}^C \left[1 + \lambda(P_t^C)\right],$$

where we set P_0^C such that the initial relative price of investment is equal to one; see below.

Appendix A.3. Neutral Technical Change

The series of neutral technical change is computed using measures of real output Y_t, real capital K_t, and labor input H_t, together with an estimate of the input shares of production. Real output Y_t is computed as the nominal gross national product, GNP_t, deflated by P_t. We convert output, capital, and hours in per capita terms dividing by civilian noninstitutional population $Pop16_t$. We explicitly consider capital quality improvement represented by the historical fall in the real price of investment. To do so, we build quarterly series for investment in efficiency units and physical depreciation rates that we use to construct series of quality-adjusted capital stock. Quality adjustments substantially change the series of capital — real capital falls below capital in efficiency units and affects the trend of neutral technical change.

Quarterly Quality-Adjusted Investment, X_t. Total investment in efficiency units is defined as total deannualized nominal investment deflated by the quality-adjusted price of investment,

$$X_t = \frac{\text{InvEQ}_t + \text{InvST}_t}{P_t^I}.$$

Quarterly Quality-Adjusted Depreciation Rates, δ_t. We build on the time-varying annual physical depreciation rates for total capital provided in Cummins and Violante (2002) for the period 1947-2000, δ_{year}^{CV}. For the years after 2000, we assume a constant depreciation rate equal to that in year 2000. We define δ_0 as the average quarterly depreciation rate over the period 1955:Q3 to 2006:Q4: $\delta_0 = 0.013$.

Quarterly Quality-Adjusted Capital Stock, K_t. We have created quarterly quality-adjusted investment series, X_t, and quarterly series for the quality-adjusted depreciation rate, δ_t. Then we can construct the series of capital in efficiency units recursively using the perpetual inventory method,

$$K_{t+1} = (1 - \delta_0)\ K_t + X_t$$

where the initial capital stock in efficiency units, K_0, is calibrated using the steady-state investment equation

$$\frac{K_0}{Y_0} = \frac{V_0\ I_0}{Y_0}\left(1 - (1 - \delta_0)\exp\left(-\lambda_K\right)\right)^{-1}.$$

We obtain the unconditional mean of the investment-output ratio is 0.284, and the quarterly capital per capita growth rate averages 1.08%. This yields an initial quarterly capital-output ratio of 11.6 (or 2.92 annually), which together with the initial value of real output pins down an initial efficient capital stock.

Neutral Technical Change, A_t. The series of neutral technical change is computed as

$$A_t = \frac{Y_t}{K_t^{\alpha}\ H_t^{1-\alpha}},$$

where $\alpha = \sum_t \frac{\alpha_t}{T}$ is the baseline capital share in Ríos-Rull and Santaeulàlia-Llopis (2010).

Appendix B. The Model

In terms of the transformed variables, the deterministic steady state of our model is characterized by the following set of equations:

$$\begin{aligned}
q^* &= e^{\frac{1}{1-\alpha}\gamma_a + \frac{\alpha}{1-\alpha}\gamma_v} \qquad \text{(B.1)}\\
v^* &= e^{\gamma_v}\\
R^* &= \frac{q^* v^*}{\beta} - (1 - \delta_0)\\
\frac{K^*}{Y^*} &= \frac{\alpha q^* v^*}{R^*}\\
\frac{X^*}{Y^*} &= \left(1 - \frac{1-\delta_0}{q^* v^*}\right)\frac{K^*}{Y^*}\\
I^* &= X^*\\
\frac{I^*}{K^*} &= 1 - \frac{1-\delta_0}{q^* v^*}\\
\frac{C^*}{Y^*} &= \frac{1}{g_*} - \frac{I^*}{Y^*}
\end{aligned}$$

For the technology shock processes, let $\hat{A}_t = \ln A_t - \ln A_0 - \gamma_a t$ and $\hat{V}_t = \ln V_t - \ln V_0 - \gamma_v t$. For other variables X_t, let $\hat{x}_t = ln(X_t/X^*)$. Then the log-linearized equilibrium conditions are given by (we scale the labor supply shock $\ln B_t$ by the factor $-\nu$ such that $\hat{b}_t = -\nu \ln B_t$):

$$
\begin{aligned}
\hat{r}_t &= \hat{y}_t - (\hat{k}_t + \hat{u}_t) + \frac{1}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{w}_t &= \hat{y}_t - \hat{h}_t \\
\hat{c}_t &= E_t[\hat{c}_{t+1}] - \frac{R^*}{R^* + 1 - \delta_0} E_t[\hat{r}_{t+1}] + \frac{1}{1-\alpha} E_t[\hat{a}_{t+1} + \hat{v}_{t+1}] \\
\hat{h}_t &= \nu(\hat{w}_t - \hat{c}_t) + \hat{b}_t \\
\hat{y}_t &= g_* \frac{C^*}{Y^*} \hat{c}_t + g_* \frac{I^*}{Y^*} \hat{i}_t + \hat{g}_t \\
\hat{y}_t &= (1-\alpha)\hat{h}_t + \alpha(\hat{k}_t + \hat{u}_t) - \frac{\alpha}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{k}_{t+1} &= \left(1 - \frac{I^*}{K^*}\right)(\hat{k}_t - R^* \hat{u}_t) + \frac{I^*}{K^*}\hat{i}_t - \frac{1 - I^*/K^*}{1-\alpha}(\hat{a}_t + \hat{v}_t) \\
\hat{u}_t &= \zeta \hat{r}_t.
\end{aligned} \tag{B.2}
$$

Notice that $\delta'(u_t) = \delta_1(1 + 1/\zeta)u_t^{1/\zeta}$ and $\delta''(u_t) = \delta_1(1+1/\zeta)(1/\zeta)u_t^{1/\zeta - 1}$. Since in steady state $R^* = \delta'(u^*)$, we deduce that $\delta''(u^*) = R^*/\zeta$, which delivers the last equation. The exogenous shock processes evolve according to

$$
\begin{aligned}
\hat{a}_t &= \hat{A}_t - \hat{A}_{t-1} \\
\hat{v}_t &= \hat{V}_t - \hat{V}_{t-1} \\
\hat{A}_t &= \psi_{1,a}(1 - \psi_{2,a})\hat{A}_{t-1} + \psi_{2,a}\hat{A}_{t-2} + \sigma_a \epsilon_{a,t} \\
\hat{V}_t &= \psi_{1,v}(1 - \psi_{2,v})\hat{V}_{t-1} + \psi_{2,v}\hat{V}_{t-2} + \sigma_v \epsilon_{v,t} \\
\hat{b}_t &= \rho_b \hat{b}_{t-1} + \sigma_b \epsilon_{b,t} \\
\hat{g}_t &= \rho_g \hat{g}_{t-1} + \sigma_g \epsilon_{g,t}.
\end{aligned} \tag{B.3}
$$

For the likelihood-based estimation of the technology shock processes and the complete DSGE models, we use the Kalman filter. Since $\hat{A}_t$ and $\hat{V}_t$ are potentially non-stationary, we initialize the filter by assuming that all hat-variables are equal to zero in period $t = -20$, where $t = 1$ corresponds to the first observation in our sample. In order to allow a marginal data density comparison between the DSGE model and the VAR, the estimation in Section 4 is based on the likelihood function that conditions on the first four sample observations ($t = 1, \ldots, 4$). The variable utilization model is estimated based on the unconditional likelihood function. Parameter estimates for our benchmark specification are tabulated in Table A-1.

Appendix C. Impulse Response to a Technology Shock

We will show that the impulse response function of labor productivity and hours worked suffices to identify the labor supply elasticity. It is apparent from (B.2) that the two technology shocks enter the system in an identical manner, at least as far as detrended output, consumption, wages, hours, capital, and the rental rate of capital are concerned. Hence, without loss of generality we will focus on the response to an investment-specific technology shock. We will assume that $\psi_{1,v} = 1$ and define $\tilde{v}_t = \hat{v}_t/(1-\alpha)$ and omit the hats from all other variables. Thus, the impulse response function has to satisfy the following equilibrium conditions:

$$\begin{aligned}
r_t &= y_t - (k_t + u_t) + \tilde{v}_t \\
w_t &= y_t - h_t \\
c_t &= E_t[c_{t+1}] - r^* E_t[r_{t+1}] + E_t[\tilde{v}_{t+1}] \\
h_t &= \nu(w_t - c_t) \\
y_t &= s_c c_t + s_i i_t \\
y_t &= (1-\alpha)h_t + \alpha(k_t + u_t) - \alpha\tilde{v}_t \\
k_{t+1} &= (1-\delta^*)(k_t - R^* u_t) + \delta^* i_t - (1-\delta^*)\tilde{v}_t \\
u_t &= \zeta r_t \\
\tilde{v}_t &= -\psi_{2,v}\tilde{v}_{t-1} + \frac{\sigma_v}{1-\alpha}\epsilon_{v,t},
\end{aligned} \tag{C.1}$$

where $r^* = R^*/(R^* + 1 - \delta_0)$, $s_c = g_* C^*/Y^*$, $s_i = g_* I^*/Y^*$, and $\delta^* = 1 - (1-\delta_0)/(q^* v^*)$. To construct the impulse response function, we assume that the system is in its steady state prior to $t = 1$, that $\epsilon_{v,1} = 1$, and $\epsilon_{v,t} = 0$ for $t > 1$. Thus, the time-path of the technology growth process is given by

$$\tilde{v}_t = (-\psi_{2,v})^{t-1}\frac{\sigma_v}{1-\alpha}, \quad E_t[\tilde{v}_{t+1}] = \tilde{v}_t. \tag{C.2}$$

After period 1 there is perfect foresight along the impulse response, and for any variable x_t it is the case that $E_t[x_{t+1}] = x_{t+1}$. With this in mind, we write the system for $t > 1$ as

$$\begin{aligned} w_t &= y_t - h_t \\ \Delta c_{t+1} &= r^* y_{t+1} - r^*(k_{t+1} + u_{t+1} - \tilde{v}_{t+1}) - \tilde{v}_{t+1} \\ h_t &= \nu(w_t - c_t) \\ y_t &= s_c c_t + s_i i_t \\ w_t &= \alpha(k_t + u_t - \tilde{v}_t) - \alpha h_t \\ k_{t+1} &= (1-\delta^*)(k_t - R^* \zeta r_t) + \delta^* i_t - (1-\delta^*)\tilde{v}_t \\ r_t &= \frac{1}{1+\zeta}(y_t - k_t + \tilde{v}_t) \end{aligned} \tag{C.3}$$

The Frisch elasticity can be obtained from the response function of wages, i.e., labor productivity, and hours worked, because it has to satisfy

$$\Delta h_{t+1} = \nu(\Delta w_{t+1} + \Delta c_{t+1}). \tag{C.4}$$

While we do not use direct information on consumption in our empirical analysis, we can deduce from (C.3) that

$$\begin{aligned} \Delta c_{t+1} &= r^* y_{t+1} - r^*(k_{t+1} + u_{t+1} - \tilde{v}_{t+1}) - \tilde{v}_{t+1} \\ &= r^*(w_{t+1} + h_{t+1}) - r^*(\alpha^{-1} w_{t+1} + h_{t+1}) - \tilde{v}_{t+1} \\ &= r^*(1-\alpha^{-1}) w_{t+1} - \tilde{v}_{t+1}. \end{aligned}$$

Thus, for $t > 1$ the impulse response function of wages and hours needs to satisfy

$$\Delta h_{t+1} = \nu \left[\Delta w_{t+1} + r^*(1-\alpha^{-1}) w_{t+1} - (-\psi_{2,v})^t \frac{\sigma_v}{1-\alpha} \right]. \tag{C.5}$$

Since r^*, α, $\psi_{2,v}$, and σ_v can be identified independently from information other than that contained in the impulse response function of hours and wages to a technology shock, we deduce that ν is identifiable as long as the initial response of hours worked to a technology shock is non-zero. Moreover, ν remains identifiable in the presence of variable capital utilization $\zeta > 0$.

Appendix D. Further Results

Table A-2 reports the full set of parameter estimates for the highest posterior probability specifications based on the data sets *Y/H, H, P*, *Y/H, H, X*, and *Y/H, H, P, X*.

Appendix E. Variable Capital Utilization

Calibration: To implement the indirect inference procedure to calibrate $\theta_{(a)} = [\psi_{1,a}, \psi_{2,a}, \sigma_a]'$, we need to construct the model implied measured TFP, which is given by

$$A_t^m = \frac{Y_t}{(K_t^m)^{\alpha} H_t^{1-\alpha}}, \tag{E.1}$$

where K_t^m stands for measured capital, that is, the economy's capital stock when abstracting from utilization dependent depreciation rates. Measured capital stock evolves as

$$K_{t+1}^m = (1-\delta) K_t^m + X_t. \tag{E.2}$$

We add equations (E.1)-(E.2) to the equilibrium conditions of our model.

The indirect inference procedure to calibrate $\theta_{(a)}$ can be described by the following steps:

1. Given $\theta_{(-a)} = [\alpha, \beta, \delta_0, \nu, \zeta, \psi_{1,v}, \psi_{2,v}, \sigma_v]'$, pick $\theta_{(a)}^s \in \mathcal{T}_{(a)}$, where $\mathcal{T}_{(a)}$ is a grid with 50,000 triplets defined as follows:

   $$[0.25, 0.265, \ldots, 1.00] \otimes [-0.2, -0.192, \ldots, 0.2] \otimes [0.001, 0.0015, \ldots, 0.01]$$

   Thus, $s = 1, \ldots, 50,000$.
2. Simulate the model 10,000 periods[26] setting $\ln A_0$ and γ_a to zero.
3. Fit an AR(2) model to the model implied measured TFP

   $$\ln A_t^m = \rho_{1,a}^m \ln A_{t-1}^m + \rho_{2,a}^m \ln A_{t-2}^m + \sigma_a^m \epsilon_t^m \tag{E.3}$$

   and estimate it using least squares. Given the sample size, we do not have to worry about the small sample effects of OLS.
4. Convert the least squares estimates of $\rho_{1,a}^m$, $\rho_{2,a}^m$, and σ_a^m into $\theta_{(a)}^m$.
5. Evaluate the discrepancy function $Q(\theta_{(a)}; \theta_{(-a)})$ at $\theta_{(a)}^m$. The discrepancy function is defined as

   $$Q(\theta_{(a)}; \theta_{(-a)}) = [\bar{\theta}_{(a),D}^m - \theta_{(a),S}^m(\theta_{(a)}, \theta_{(-a)})]' \bar{V}_{(a)}^{-1} [\bar{\theta}_{(a),D}^m - \theta_{(a),S}^m(\theta_{(a)}, \theta_{(-a)})].$$

   We used the additional subscripts D and S to denote estimates computed based on the actual and simulated data, respectively. In fact, $\bar{\theta}_{(a),D}^m$ corresponds to the posterior means reported in Table 1, and $\bar{V}_{(a)}$ is the posterior covariance matrix.

[26] We simulate the model economy for 10,200 periods and discard the first 200.

6. If $s < 50,000$, go to step 1. Otherwise, compute

$$\hat{\theta}_{(a)} = \text{argmin}_{\theta_{(a)} \in \mathcal{T}_{(a)}} Q(\theta_{(a)}; \theta_{(-a)}),$$

Table A-1: POSTERIOR ESTIMATES FOR BENCHMARK SPECIFICATION

Series	Y/H, H, P	
Shocks	A, V, B	
Unit Root	No	
α	0.361	[0.326, 0.395]
ν	0.852	[0.344, 1.326]
γ_a	-0.002	[-0.004, 0.000]
$\psi_{1,a}$	0.983	[0.973, 0.993]
$\psi_{2,a}$	-0.103	[-0.242, 0.038]
σ_a	0.007	[0.007, 0.008]
γ_v	0.007	[0.007, 0.008]
$\psi_{1,v}$	0.990	[0.986, 0.993]
$\psi_{2,v}$	-0.714	[-0.796, -0.632]
σ_v	0.003	[0.003, 0.004]
ρ_b	0.968	[0.952, 0.986]
σ_b	0.012	[0.010, 0.014]
$\ln H^*$	-0.037	[-0.072, -0.004]
$\ln Y_0$	9.137	[8.628, 9.692]
$\ln V_0$	-0.095	[-0.153, -0.037]

Note: The following parameters are fixed during the estimation: $\beta = 0.99$ and $\delta_0 = 0.013$.

Table A-2: POSTERIOR ESTIMATES FOR HIGHEST POST. PROB. SPECIFICATIONS

Series	Y/H, H, P		Y/H, H, X		Y/H, H, P, X	
Shocks	A, V, G		A, V, B, G		A, V, B, G	
Unit Root	Yes		Yes		No	
α	0.340	[0.306, 0.374]	0.323	[0.292, 0.353]	0.391	[0.381, 0.402]
ν	0.419	[0.168, 0.643]	0.964	[0.328, 1.539]	0.170	[0.048, 0.284]
γ_a	0.000	[-0.001, 0.001]	0.000	[-0.001, 0.001]	-0.001	[-0.001, -0.001]
$\psi_{1,a}$	1.000		1.000		0.950	[0.931, 0.968]
$\psi_{2,a}$	-0.020	[-0.148, 0.120]	-0.006	[-0.089, 0.073]	-0.088	[-0.203, 0.029]
σ_a	0.007	[0.006, 0.008]	0.007	[0.007, 0.008]	0.007	[0.007, 0.008]
γ_v	0.007	[0.006, 0.008]	0.007	[0.006, 0.008]	0.008	[0.007, 0.008]
$\psi_{1,v}$	1.000		1.000		0.991	[0.988, 0.994]
$\psi_{2,v}$	-0.694	[-0.769, -0.620]	-0.059	[-0.140, 0.025]	-0.646	[-0.722, -0.570]
σ_v	0.003	[0.003, 0.004]	0.007	[0.006, 0.008]	0.003	[0.003, 0.004]
ρ_b			0.967	[0.951, 0.983]	0.953	[0.935, 0.970]
σ_b			0.011	[0.009, 0.013]	0.009	[0.008, 0.010]
ρ_g	0.962	[0.944, 0.982]	0.972	[0.952, 0.993]	0.963	[0.949, 0.978]
σ_g	0.038	[0.021, 0.056]	0.004	[0.003, 0.006]	0.010	[0.008, 0.011]
$\ln H^*$	-0.028	[-0.067, 0.009]	-0.024	[-0.064, 0.012]	-0.027	[-0.049, -0.004]
$\ln Y_0$	-32.284	[-49.319, -17.905]	8.377	[4.489, 12.753]	8.627	[8.548, 8.704]
$\ln V_0$	27.552	[17.036, 41.493]	-0.044	[-3.062, 2.724]	-0.148	[-0.229, -0.066]

Note: The following parameters are fixed during the estimation: $\beta = 0.99$, $\delta_0 = 0.013$, and $g^* = 1.2$ (in models with G-shock).