Online Appendix: "Countercyclical Elasticity of Substitution", by Dongya Koh and Raül Santaeulàlia-Llopis

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A Data Construction

This appendix provides the sources of data in Section A.1, and replicates the construction details of intensive/extensive margins of labor supply in Section A.2, the labor share in Section A.3, the relative price of investment in Section A.4, quality-adjusted investment and capital stock in Section A.5, neutral technical changes in Section A.6, and labor-augmenting technical changes in Section A.7.

A.1 Raw Data Series

All raw data series were retrieved from the Bureau of Economic Analysis (BEA; www.bea.gov) and the Bureau of Labor Statistics (BLS; www.bls.gov) for the period 1948.I–2019.IV. All series are retrieved as seasonally adjusted.

National Income and Product Accounts (NIPA-BEA)

1. Table 1.1.5: Consumption of Durable Goods (CD_t), Change in Inventories ($ChInv_t$)
2. Table 1.1.6: Personal Consumption Expenditures (PCE_t)
3. Table 1.7.5: Gross National Product (GNP_t), (Private) Consumption Fixed Capital (DEP_t), Statistical Discrepancy ($SDis_t$)
4. Table 1.12: Compensation of Employees (CE_t), Proprietor's Income (PI), Rental Income (RI), Corporate Profits (CP), Net Interests (NI), Taxes on Production (Tax), Subsidies (Sub), Business Current Transfer Payments (BCTP), Current Surplus of Government Enterprise (GE)
5. Tables 2.3.3 and 2.3.5: Quantity Index ($QCONS_t^i$), Nominal ($CONS_t^i$) Nondurables Consumption (excluding gasoline and other energy goods) and Services (excluding housing and utilities)[1]
6. Table 3.9.5: Government Investment in Equipment($GovIEQ_t$), Government Investment in Structures($GovIST_t$)
7. Table 5.3.4: Official Price Index for Investment in Equipment for Residential and Non-residential($OPIEQ_t$)
8. Table 5.3.5: Private Fixed Investment in Equipment for Residential and Non-residential($PrivIEQ_t$), Private Fixed Investment in Structures for Residential and Non-residential($PrivIST_t$)
9. Table 5.8.5 Private Inventories (Inv)

Bureau of Labor Statistics (BLS-CES,BLS-CPS)

1. Average Weekly Hours (AWH_t), Nonfarm Business Sector, Index 2009=100, Series ID PRS85006023
2. Employment Level $+16$ (EMP_t), Nonfarm Business Sector, Index 2009=100, Series ID PRS85006013
3. Civilian Noninstitutional Population $+16$ (POP_t), In Thousands, Series ID LNS10000000
4. Civilian Labor Force $+16$ (LAB_t), In Thousands, Series ID LNS11000000

As in Ríos-Rull et al. (2012), we use GNP_t deflated with a quarterly price index for consumption, P_t^C (see below), divided by the working-age population $Pop16_t$ to construct output per capita. We construct aggregate hours, average hours of work and employment rate from BLS index of average weekly hours h_t (see below).

[1] Goods i correspond to nondurables consumption in food, clothing and shoes and others, and services in health care, transportation, recreation, food services and accommodations, financial services and insurance and others.

A.2 Intensive and Extensive Margins of Labor Supply

We construct the series using average weekly hours worked per worker (AWH_t), number of workers (EMP_t), noncivilian population (POP_t) and noncivilian labor force (LAB_t) in the following way:

1. Average hours per worker, $\text{h}_t = \text{AWH}_t$
2. Aggregate hours, $\text{L}_t = \text{h}_t \times \text{EMP}_t$
3. Employment per capita, $\text{e}_t = \text{EMP}_t/\text{LAB}_t \times \text{LAB}_t/\text{POP}_t = \text{EMP}_t/\text{POP}_t$
4. Hours per capita, $\text{eh}_t = \text{L}_t/\text{POP}_t = \text{e}_t \times \text{h}_t$

where employment per capita e_t is the product of the employment rate $\text{EMP}_t/\text{LAB}_t$ times the labor force participation rate $\text{LAB}_t/\text{POP}_t$. Note that the standard RBC definition of hours per capita, L_t/POP_t, is eh_t in our model. The variance of hours per capita can be decomposed as $var(eh_t) = var(h_t) + var(e_t) + 2cov(h_t, e_t)$.[2]

A.3 Labor Share

To construct labor share we use only nominal variables and follow Ríos-Rull and Santaeulàlia-Llopis (2010):

1. Unambiguous Capital Income (UCI) = RI + CP + NI + GE
2. Unambiguous Income (UI) = UCI + DEP + CE
3. Proportion of Unambiguous Capital Income to Unambiguous Income = $\theta_P = \frac{\text{UCI+DEP}}{\text{UI}}$. Then we can use θ_P to compute the amount of ambiguous capital income in ambiguous income,
4. Ambiguous Income (AI) = PI + Tax - Sub + BCTP + SDis
5. Ambiguous Capital Income (ACI) = $\theta_P \times$ AI.

Then, capital income (service flows of private fixed capital), Y_{KP}, is computed as the sum of unambiguous capital income, depreciation, and ambiguous capital income; that is,

$$Y_{KP} = \text{UCI} + \text{DEP} + \text{ACI}$$

which we use to construct out baseline labor share as

$$\text{Labor Share} = 1 - \frac{\text{UCI} + \text{DEP} + \text{ACI}}{\text{GNP}} = 1 - \frac{Y_{KP}}{\text{GNP}} = 1 - \theta_P.$$

[2] Note that employment per capita e is the product of the employment rate (number of workers divided by non-civilian labor force) times the participation rate (non-civilian labor force divided by non-civilian population). Alternatively, we could have chosen to set the participation rate to be equal to one (or to a constant) and ignored the contribution of fluctuations in labor force to fluctuations in hours per capita. In the case that the participation rate is one, employment per capita is the employment rate. However, while we find that the variance of employment per capita is 2.76, the variance of its component is 2.32 for the employment rate and .14 for the participation rate. This implies a small positive covariance of .15 between the employment rate and the participation rate. Splitting the contribution of the covariance equally between the employment rate and the participation rate implies that fluctuations in the employment rate account for (2.32+.15/2)×100/2.76=89% of the variance of employment per capita, while the participation accounts for 11%. Therefore, quantitatively we do not find much difference between explicitly incorporating the participation rate and not incorporating. We decided to keep our definition of employment per capita as the product of the employment rate and the participation rate.

A.4 The Relative Price of Investment

The construction of the relative price of (quality-adjusted) investment, P_t^I, closely follows Ríos-Rull et al. (2012). P_t^I is built as a Tornqvist aggregate of the price index of quality-adjusted equipment investment and the price index of structures investment. The price index of consumption, P_t^C, is used as a proxy for the price of structures investment.[3] Based on P_t^I and P_t^C, the relative price of investment (using the consumption as numeraire) is defined as

$$P_t = \frac{P_t^I}{P_t^C}.$$

Its inverse, $V_t = \frac{1}{P_t}$, is ISTC.

Quarterly Quality-Adjusted Price Index for Investment in Equipment, QAPIEQ$_t$. The U.S. 1947–2000 annual series provided by Cummins and Violante (2002) are used for the price index of equipment investment, QAPIEQ$_{year}^{CV}$. Then, quarterly movements of NIPA-BEA price index of equipment investment, OPIEQ$_t$, are imputed to the annual series using the Denton method. In computing the price index of equipment investment, the price index for resident and non-resident are weighted by the share of nominal quantity (PrivIEQ$_t$) for resident and non-resident, even though the equal weights do not necessarily affect the aggregate price index. Further, while in Ríos-Rull et al. (2012) we use the official price index OPIEQ$_t$ for the years after 2000.[4,5] This is not necessary for the current exercise, as here the sample is restricted to 1955.I–2000.IV for comparison purposes with Fisher (2006).

Quarterly Quality-Adjusted Price Index for Total Investment, P_t^I. We use a Tornquist price index aggregate that weights growth rates of the price index of investment in equipment and the price index of investment in structures by their nominal shares, s_t^{IEQ} and s_t^{IST}. Nominal equipment investment is the sum of private equipment investment (PrivIEQ$_t$), government equipment investment (GovIEQ$_t$), changes in inventories (ChInv$_t$), and consumer durables (CD$_t$). Nominal structures investment is the sum of private structures investment (PrivIST$_t$) and government structures investment (GovIST$_t$). The growth rate of the quarterly quality-adjusted price index for total investment is

$$\lambda(P_t^I) = \left(\frac{s_t^{IEQ} + s_{t-1}^{IEQ}}{2}\right)\lambda(\text{QAPIEQ}_t) \; + \; \left(\frac{s_t^{IST} + s_{t-1}^{IST}}{2}\right)\lambda(P_t^C),$$

where $\lambda(x_t) = (x_t - x_{t-1})/x_t$ and changes in the price index for consumption goods, $\lambda(P_t^C)$, serve as a proxy for inflation in the price of structures. The level of the quarterly quality-adjusted price index for total investment is recovered recursively,

$$P_t^I = P_{t-1}^I[1 + \lambda(P_t^I)].$$

We use the initial value P_0^I suggested in Cummins and Violante (2002).

Quarterly Price Index for Consumption, P_t^C. We use a Tornquist price index aggregate that weights growth rates of price indexes for nondurables consumption (food, clothing and shoes, and others) and services (household operations, transportation, medical care, recreation, and others) by their nominal shares. Let $P_t^{C,i}$ be the price index for nondurable consumption/service good i in quarter t computed as the ratio between nominal consumption

[3] As is standard in the previous literature, we use the consumption deflator as the price index for investment in structures (see Fisher (2006) and Canova et al. (2010)). This provides internal consistency in how we compute the quality-adjusted price index for total (equipment + structures) investment—one of the elements of output is investment; hence, the alternative use of an output (instead of a consumption) deflator potentially distorts the investment deflator.

[4] Rescaled such that it equates to the value in Cummins and Violante (2002) in the year 2000.

[5] That implicitly assumes that the hedonic methods used to compute the official price index will correctly quality-adjust most types of equipment investment after 2000.

of good i, CONS_t^i, and the quantity index of good i, QCONS_t^i. Let s_t^i be the corresponding nominal share of good i in period t. Then, the growth rate of the price index for consumption is

$$\lambda(P_t^C) = \sum_i \frac{s_t^i + s_{t-1}^i}{2} \lambda(P_t^{C,i}),$$

The level of the consumption price index is recovered recursively,

$$P_t^C = P_{t-1}^C \left[1 + \lambda(P_t^C)\right]$$

where we set P_0^C such that the initial relative price of investment is equal to 1.

A.5 Quality-Adjusted Investment and Capital Stock

Quarterly Quality-Adjusted Investment, X_t. Total investment in efficiency units is defined as total de-annualized nominal investment deflated by the quality-adjusted price of investment,

$$X_t = \frac{InvEQ_t + invST_t}{P_t^I}$$

where nominal equipment investment is constructed as $\text{InvEQ}_t = \text{PrivIEQ}_t + \text{GovIEQ}_t + \text{ChInv}_t + \text{CD}_t$ and nominal structures investment as $\text{invST}_t = \text{PrivIST}_t + \text{GovIST}_t$. Given the quality-adjusted investment series, we generate $I_t = P_t X_t = \frac{X_t}{V_t}$.

Quarterly Quality-Adjusted Capital Stock, K_t. We have created quarterly quality-adjusted investment series, X_t. Then we can construct the series of capital in efficiency units recursively using the perpetual inventory method,

$$K_{t+1} = (1 - \delta_0)K_t + X_t$$

where long-run capital depreciation rate (δ_0) is calibrated as in the context, and the initial capital stock in efficiency units, K_0, is calibrated using the steady-state investment equation,

$$\frac{K_0}{Y_0} = \frac{V_0 I_0}{Y_0}(1 - (1 - \delta_0)(1 + \lambda_k))^{-1}.$$

We obtain the unconditional mean of the investment-output ratio as 0.28; the quarterly capital per capita growth rate averages 1.11% and quarterly population growth rate averages 0.25%. This yields an initial quarterly capital-output ratio of 10.5013, which together with the initial value of real output pins down an initial efficient capital stock.

A.6 Productivity Residual

We construct the productivity residual (neutral technical change) from the standard growth accounting equation. Following Solow (1957), we recover the growth rate of Solow residual from the equation below.

$$\gamma_{z,t} = \gamma_{y,t} - (1 - ls_t)\gamma_{k,t} - ls_t \gamma_{eh,t} \tag{1}$$

where $\gamma_x = \frac{x_t - x_{t-1}}{x_{t-1}}$ is the growth rate of variable x. The data for y indicates per capita real GNP, eh standard RBC average hours of work, k the quality-adjusted per capita stock of capital with constant depreciation, and ls labor share, all of which are constructed in previous sections. The construction of neutral technical change implicitly takes into account the presence of investment-specific technical change but not the capital utilization. Given a series of $\gamma_{z,t}$, we recover neutral technical change by assuming initial technical change in 1948 to be

$z_{1948} = 1$.[6]

A.7 Identification of Labor-Augmenting Technical Change

In order to recover the short-run component of labor-augmenting technical change—i.e. deviations from trend, we totally differentiate production evaluated at the steady state. First, we write our NCES-CD production technology (dropping time subscripts) as:

$$y\left(x\right) = \left(\left(1-\alpha\right) \left(\frac{k}{k^*}\right)^{\frac{\sigma-1}{\sigma}} + \alpha \left(\frac{al}{a^* l^*}\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} y^*$$

with vector $x = (k, al, \sigma)$ and generic element x_i. That is, production can deviate from steady state due to factor input choices, including capital k and labor l, labor augmenting technical change, a, and the short-run elasticity between capital and labor, σ. Note that we can write $y(x) = \underbrace{b(x)^{g(x)}}_{y(x)/y^*} y^*$ where the base $b(x)$ and the exponent $g(x)$ of short-run production are:

$$b(x) = \left(\left(1-\alpha\right) \left(\frac{k}{k^*}\right)^{\frac{\sigma-1}{\sigma}} + \alpha \left(\frac{al}{a^* l^*}\right)^{\frac{\sigma-1}{\sigma}} \right)$$
$$g(x) = g(\sigma) = \frac{\sigma}{\sigma-1}$$

Let's denote the level difference with respect to steady state (a balanced growth) as $dx = x - x^*$ and the rate difference as $\gamma_x = \frac{dx}{x^*}$ where x^* is the steady state.

Second, we totally differentiate $y(x) = b(x)^{g(x)}$ evaluated at steady state:[7]

$$\frac{dy(x)}{y^*} = g(x) b(x)^{g(x)-1} \sum_{x_i \in x} \frac{\partial b(x)}{\partial x_i}\Big|_{x=x^*} (x_i - x_i^*) + b(x)^{g(x)} ln b(x) \sum_{x_i \in x} \frac{\partial g(x)}{\partial x_i}\Big|_{x=x^*} (x_i - x_i^*) \tag{2}$$

where note that $\frac{\partial g(x)}{\partial x_i}$ is nonzero only for $x_i = \sigma$, which implies that we can rewrite (2) as:

$$\frac{dy(x)}{y^*} = g(x) b(x)^{g(x)-1} \sum_{x_i \in x} \frac{\partial b(x)}{\partial x_i}\Big|_{x=x^*} (x_i - x_i^*) + b(x)^{g(x)} ln b(x) \frac{\partial g(x)}{\partial \sigma}\Big|_{x=x^*} (\sigma_i - \sigma_i^*) \tag{3}$$

Then, note that at the steady state $b(x) = 1$ (i.e. $\ln b(x)\Big|_{x=x^*}$ is zero) which implies that the second component of the right hand side of (3) is zero since $y(x) = b(x)^{g(x)} \neq 0$ and $\frac{\partial g(x)}{\partial \sigma} \neq 0$. Hence,

$$\frac{dy(x)}{y^*} = g(x) b(x)^{g(x)-1} \sum_{x_i \in x} \frac{\partial b(x)}{\partial x_i}\Big|_{x=x^*} (x_i - x_i^*). \tag{4}$$

[6] Note that this object is different from labor-augmented technical change in the model, a_t, and therefore, we construct the counterpart of z_t from the model simulation by assuming the presence of ISTC and a constant depreciation.

[7] Remember that if $y(x) = b^{g(x)}$, then $dy(x) = \frac{\partial b^{g(x)}}{\partial x} dx = b^{g(x)} ln b \frac{\partial g(x)}{\partial x} dx$. Also, if $y(x) = b(x)^{e(x)}$, then $dy(x) = \frac{\partial b(x)^{g(x)}}{\partial x} dx = g(x) b(x)^{g(x)-1} \frac{\partial b(x)}{\partial x} dx + b(x)^{g(x)} ln b(x) \frac{\partial g(x)}{\partial x} dx$.

To complete the differentiation we need to develop $\frac{\partial b(x)}{\partial x_i}$ for each element of x:

$$\frac{\partial b(x)}{\partial k} = (1-\alpha)g(x)^{-1}\left(\frac{k}{k^*}\right)^{g(x)^{-1}-1}\frac{1}{k^*}$$
$$\frac{\partial b(x)}{\partial al} = \alpha g(x)^{-1}\left(\frac{al}{a^*l^*}\right)^{g(x)^{-1}-1}\frac{1}{a^*l^*}$$
$$\frac{\partial b(x)}{\partial \sigma} = \left((1-\alpha)\left(\frac{k}{k^*}\right)^{g(x)^{-1}}\ln\left(\frac{k}{k^*}\right)+\alpha\left(\frac{al}{a^*l^*}\right)^{g(x)^{-1}}\ln\left(\frac{al}{a^*l^*}\right)\right)\frac{\partial g(x)^{-1}}{\partial \sigma}$$

Here, note that that at the steady state $\ln\frac{k}{k^*}\Big|_{x=x^*} = \ln\frac{al}{a^*l^*}\Big|_{x=x^*} = 0$. That is, at the steady state $\frac{\partial b(x)}{\partial \sigma}$ equals zero. Hence, we can write

$$\begin{aligned}\frac{dy(x)}{y^*} &= g(x)b(x)^{g(x)-1}(1-\alpha)g(x)^{-1}\left(\frac{k}{k^*}\right)^{g(x)^{-1}-1}\frac{1}{k^*}\Big|_{x=x^*}(k-k^*)\\ &+ g(x)b(x)^{g(x)-1}\alpha g(x)^{-1}\left(\frac{al}{a^*l^*}\right)^{g(x)^{-1}-1}\frac{1}{a^*l^*}\Big|_{x=x^*}(al-a^*l^*)\end{aligned}$$

which after a bit algebra implies,

$$\begin{aligned}\frac{dy(x)}{y^*} &= (1-\alpha)\left(\frac{\frac{k}{k^*}}{\frac{y}{y^*}}\right)^{-\frac{1}{\sigma}}\frac{1}{k^*}\Big|_{x=x^*}(k-k^*)+\alpha\left(\frac{\frac{al}{a^*l^*}}{\frac{y}{y^*}}\right)^{-\frac{1}{\sigma}}\frac{1}{a^*l^*}\Big|_{x=x^*}(al-a^*l^*)\\ &= (1-\alpha)\frac{(k-k^*)}{k^*}+\alpha\frac{(l-l^*)}{l^*}+\alpha\frac{(a-a^*)}{a^*}\end{aligned}$$

or, simply

$$\gamma_y = (1-\alpha)\gamma_k + \alpha\gamma_l + \alpha\gamma_a \tag{5}$$

In this manner, with information on the log deviations from steady state for output γ_y, capital γ_k and labor γ_l and assuming competitive markets so that the steady state labor share is α, we can recover γ_a from (5). That is, we can independetly identify $\frac{a}{a^*}$ without information about σ.

B Labor Market Cyclical Behavior and Dynamics

B.1 Cyclical Behavior of the Labor Market

Using the data constructed in Section A, we report variances and correlations of labor market variables with confidence intervals using block bootstrap with replacement on the time series data. Also, we show the IRFs of TFP, labor share, and output in response to one standard deviation TFP shock.

B.2 Estimating the IRF of Labor Share to Productivity Shocks

We follow the analysis in Ríos-Rull and Santaeulàlia-Llopis (2010) to explore the response of labor share to productivity shocks for a larger sample. We pose a trivariate autoregressive (AR) process for productivity, labor share and output for the sample period 1948.I-2019.IV.[8] We identify productivity shocks as the observable short-run log-deviations from trend from the total factor productivity. Specifically, we compute the total factor productivity residual (z_t) as in Appendix A.6:

$$\gamma_{z,t} = \gamma_{y,t} - (1 - ls_t)\gamma_{k,t} - ls_t\gamma_{eh,t}, \tag{6}$$

where $\gamma_{x,t} = \frac{x_t - x_{t-1}}{x_{t-1}}$ is the growth rate of x_t and the series of capital (k_t) is recovered by using the perpetual inventory methods,

$$i_t = k_{t+1} - (1 - \delta_0)k_t. \tag{7}$$

where i_t is net investment and δ_0 is a constant capital depreciation rate. Then, normalizing the initial value z_0 to one, we can recover the entire series z_t. We need our series to be stationary for estimation. While labor share shows no discernible trend, the series of output and productivity are logged and linearly detrended before estimation. Our trivariate autoregressive process is specified as follows,

$$\begin{aligned} z_t &= \Psi_{z,z}(L)z_t + \epsilon_{z,t}, \\ ls_t &= \Psi_{ls,ls}(L)ls_t + \Psi_{z,ls}(L)z_t + \epsilon_{ls,t}, \\ y_t &= \Psi_{y,y}(L)y_t + \Psi_{z,y}(L)z_t + \epsilon_{y,t}, \end{aligned} \tag{8}$$

where $\Psi_{k,\ell}(L) \equiv \rho_{1,k,\ell}L + \rho_{2,k,\ell}L^2$ indicates lagged marginal effects of k on ℓ, and there are two such lags with

$$\boldsymbol{\epsilon}_t = (\epsilon_{z,t}, \epsilon_{ls,t}, \epsilon_{y,t})' \sim N(0, \Sigma_{z,ls,y}), \tag{9}$$

which we factorize a la Cholesky to yield,

$$\boldsymbol{\epsilon}_t = \Omega_{z,ls,y}\boldsymbol{u}_t \quad \text{and} \quad \Omega_{z,ls,y} = \begin{pmatrix} \omega_{z,z} & 0 & 0 \\ \omega_{z,ls} & \omega_{ls,ls} & 0 \\ \omega_{z,y} & 0 & \omega_{y,y} \end{pmatrix}, \tag{10}$$

where $\boldsymbol{u}_t$ has unit variance. The identification assumptions to pin down $\Omega_{z,ls,y}$ are that innovations to productivity affect the labor share (i.e., $\omega_{z,ls} \neq 0$) but not the other way around (i.e., $\omega_{ls,z} = 0$), which is the identification assumption in Ríos-Rull and Santaeulàlia-Llopis (2010). This implies that innovations to labor share are purely redistributive. Further we also set an analogous identification for the joint dynamics between productivity and output; that is, $\omega_{z,y} \neq 0$ and $\omega_{ls,y} = 0$ and the mixed innovations $\omega_{y,ls} = \omega_{ls,y} = 0$.

[8] Originally, Ríos-Rull and Santaeulàlia-Llopis (2010) pose a bivariate process for productivity and labor share. The introduction of output in the autoregressive process does not change the results under the purely redistributive assumption of labor share shocks, that is, labor share has no contemporaneous impact on productivity or output.

Table B-1: Cyclical Behavior of the Labor Market, U.S. 1948.I-2019.IV

(a) Variance

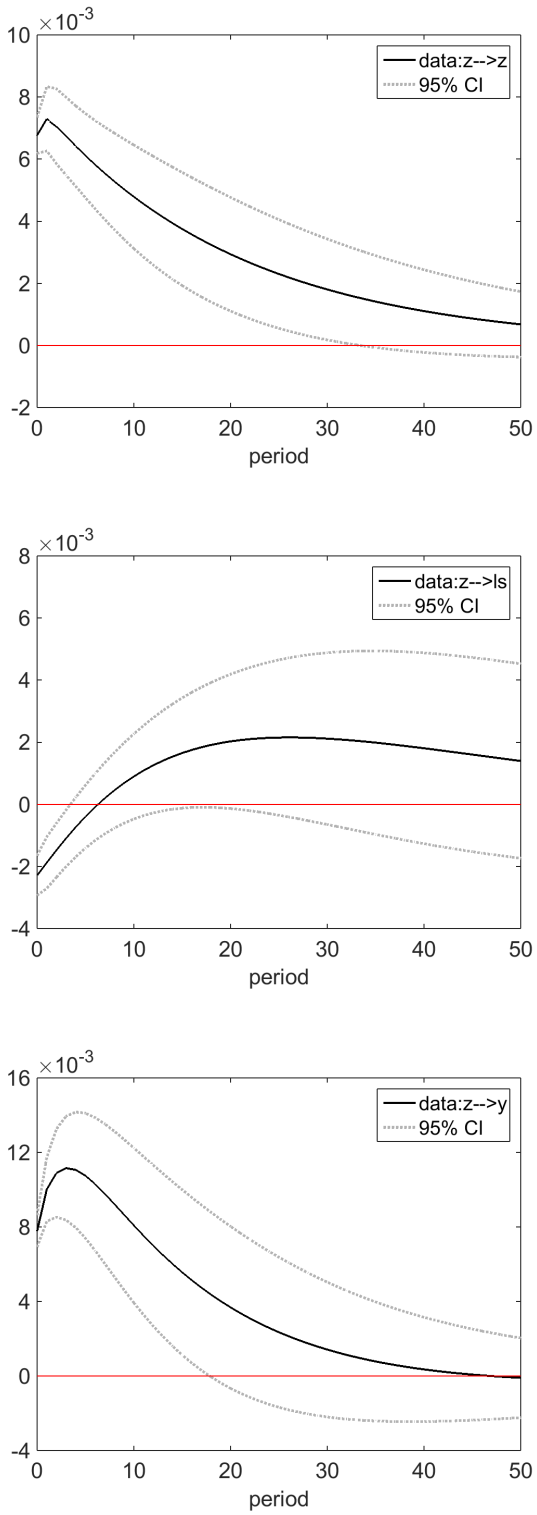
Variable	y	eh	e	h	w	lp	ls
	3.31	3.64	2.51	0.28	0.84	0.81	0.55
	[2.60, 4.03]	[2.85, 4.44]	[1.90, 3.11]	[0.25, 0.31]	[0.57, 1.10]	[0.30, 1.33]	[0.45, 0.65]

(b) Correlation Matrix

Variable	y	eh	e	h	w	lp	ls
y	1.00	0.88	0.82	0.73	**-0.13**	**0.15**	**-0.34**
		[0.81, 0.96]	[0.72, 0.92]	[0.66, 0.80]	[-0.41, 0.15]	[-0.07, 0.36]	[-0.51, -0.18]
eh		1.00	0.97	0.70	**-0.44**	**-0.33**	-0.14
			[0.96, 0.98]	[0.64, 0.76]	[-0.59, -0.29]	[-0.48, -0.19]	[-0.35, 0.07]
e			1.00	0.51	-0.41	-0.39	-0.03
				[0.41, 0.61]	[-0.59, -0.24]	[-0.54, -0.25]	[-0.28, 0.22]
h				1.00	-0.36	-0.02	-0.42
					[-0.58, -0.14]	[-0.17, 0.13]	[-0.60, -0.24]
w					1.00	0.67	0.42
						[0.50, 0.83]	[0.09, 0.76]
lp						1.00	-0.39
							[-0.68, -0.10]
ls							1.00

Notes: y denotes output per capita, $eh = H/N$ is hours per capita, e is employment per capita, h is average hours per worker, w is real wage, lp indicates labor productivity and ls is labor share. See Online Appendix A for data definitions and variable construction. For the computations of these statistics all time series have been logged and HP-filtered. Confidence intervals are computed using block bootstrap with replacement on the actual data (Lee and Ingram, 1991; Bloom, 2009).

Figure B-1: Data IRFs of TFP, Labor Share and Output to TFP Shocks



Notes: Data IRFs of TFP (top panel), labor share (center panel) and output (bottom panel) in logs in response to one standard deviation TFP shocks. We follow the identification strategy in Ríos-Rull and Santaeulàlia-Llopis (2010).

C Cross-Industry Variation in σ and the Business Cycle

We described the cyclical behavior of the aggregate elasticity of substitution using cross-time variation in Section 2.2 in the main text. An alternative procedure for describing the potential behavior of the aggregate elasticity of substitution over time is to use cross-industry variation. To do so, we assume that each industry operates a CES technology with an elasticity between capital and labor, σ_i, which is possibly different across industries. In this scenario, we can study the cyclical behavior of the industrial composition of output together with the industry-specific elasticity of substitution, σ_i. Does the share of industries with low (high) elasticity of substitution increase (decrease) in expansions? If yes, this would suggest the notion of a countercyclical aggregate σ_t, which is consistent with our structural results.[9]

Table C-1 shows the industry-specific elasticity of substitution for each of the 35 sectors in KLEM database provided in Jorgenson (2008). For this estimation we closely follow Young (2013).[10] In Table C-1, we list 32 industries with the estimated σ_i, industrial shares of output, and cyclical correlations to aggregate output.[11] The table is sorted by the value of the elasticity of σ_i from the lowest to the highest. We observe that industries with low elasticity σ_i do not present a clear pattern in terms of the cyclicality of their industrial shares with aggregate output. For example, electrical machinery has an elasticity equal to .02 and displays a positive share correlation with aggregate output of .48, while food that has also a very low elasticity of .03 shows a negative correlation with output of -.57. However, we find that industries with high elasticity, in particular, those with a larger share of aggregate output (i.e., finance and services), display clear and strong negative correlations between their respective shares of industrial output and aggregate output. To gain a better idea of this relationship, Figure C-1 shows the scatter plot of industry-specific elasticities of substitution (vertical axis) and the cyclical correlation between industry-specific shares with aggregate output. Our findings suggest an unambiguous negative relationship between industry-specific elasticities of substitution σ_i and the comovement of their respective industrial shares s_{it} with output y_t, i.e. $\rho(s_{it}, y_t)$. We obtain a significantly negative (weighted by share) correlation of -.262 between σ_i and $\rho(s_{it}, y_t)$. That is, the share of industries with a low (high) elasticity of substitution tends to increase (decrease) with output. This suggests again an aggregate elasticity of substitution that declines in expansions.

[9] There is an obvious caveat in the inference of this exercise, and that is that the addition of sectoral CES technologies (one per industry) does not imply an aggregate technology. It is, however, still useful to, at least qualitatively, see which industries (high vs. low elasticity) are more important during recessions and expansions. If the share of value added of industries with low (high) σ_i increases (decreases) in expansions, this notion would generally conform with the results of our structurally estimated σ_t, that is, that σ_t is countercyclical.

[10] Young (2013) estimates first-order conditions for optimal factor demand using several techniques, GMM, GIV, and differenced OLS. Here we focus on a standard specification with relative factor demands,

$$\frac{k_{ti}}{h_{ti}} = c_3 + \sigma_i \frac{w_{ti}}{r_{ti}} + (1 - \sigma_i)(\lambda_h - \lambda_k) \cdot t + \varepsilon_{3ti},$$

that we use to estimate σ_i separately for each industry i with OLS. Our results are in Table C-1, and they are similar to those reported in Young (2013).

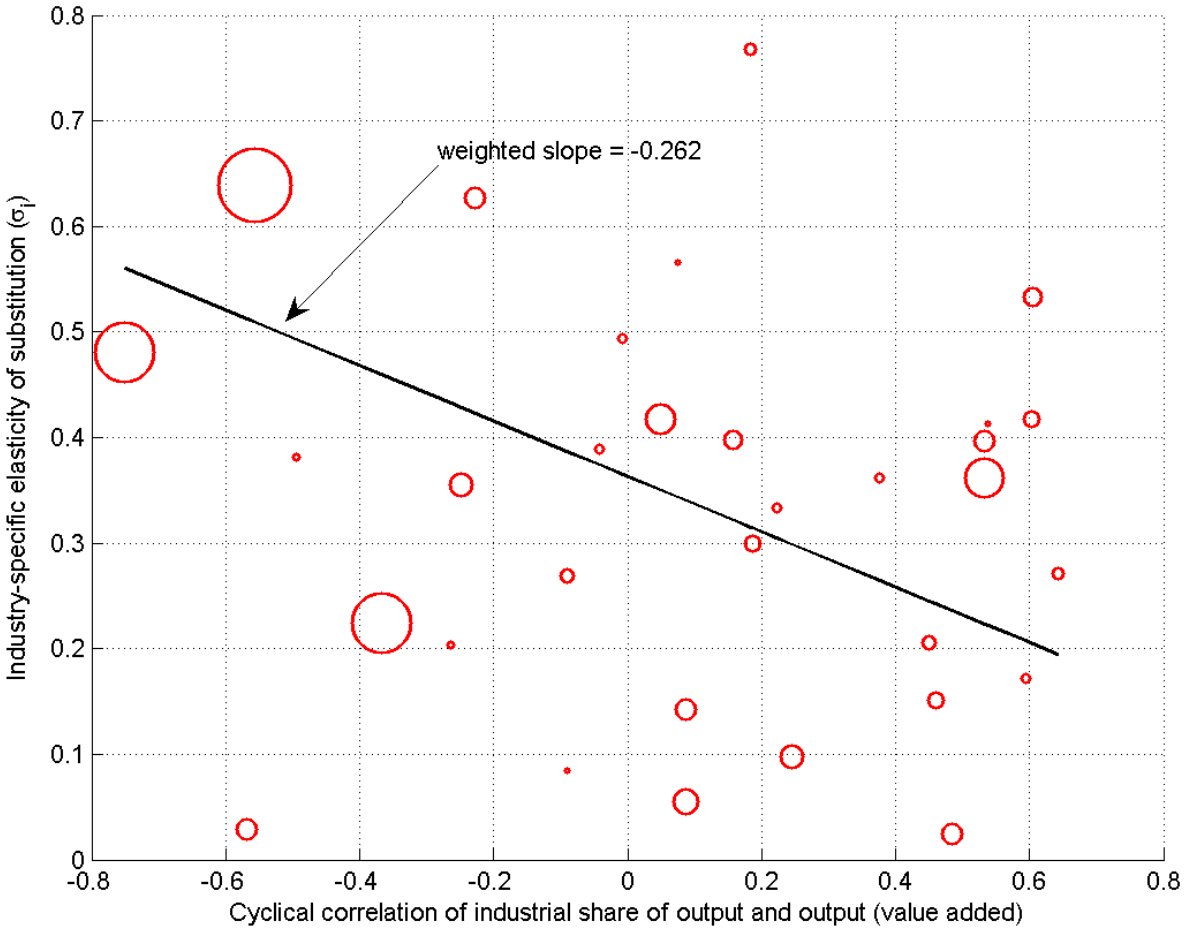
[11] We dropped three industries for which the estimated σ_i was negative. These industries are communications, agriculture, and textile mill products, which represent a relatively minor share of aggregate output, 5.1%.

Table C-1: Industry-Specific Elasticity of Substitution, σ_i, vs. Cyclical Correlation of Industrial Shares and Aggregate Output, KLEM 1960-2005

Industry Name	Elasticity σ_i	Industrial Share $s_i = \sum_t \frac{y_{i,t}/y_t}{T}$	Cyclical Correlation $\rho(s_{i,t}, y_t)$
Electrical machinery	0.02	1.51	0.48
Food and kindred products	0.03	2.17	-0.57
Government enterprises	0.06	2.76	0.09
Leather	0.09	0.15	-0.09
Chemicals	0.10	2.71	0.25
Printing, publishing and allied products	0.14	1.97	0.09
Motor vehicles	0.15	1.22	0.46
Furniture and fixtures	0.17	0.48	0.59
Coal mining	0.20	0.28	-0.26
Rubber and misc plastics	0.21	0.82	0.45
Trade	0.22	14.74	-0.37
Paper and allied products	0.27	1.12	-0.09
Stone, clay, glass	0.27	0.83	0.64
Instruments	0.30	1.19	0.19
Apparel	0.33	0.58	0.22
Electric utilities	0.36	2.65	-0.25
Petroleum and coal products	0.36	0.55	0.38
Construction	0.36	7.48	0.53
Tobacco	0.38	0.29	-0.50
Misc manufacturing	0.39	0.42	-0.04
Machinery, non-electrical	0.40	1.87	0.53
Transportation equipment and ordnance	0.40	1.70	0.16
Non-metallic mining	0.41	0.21	0.54
Transportation	0.42	4.03	0.05
Primary metal	0.42	1.36	0.60
Finance insurance and real estate	0.48	14.57	-0.75
Gas utilities	0.49	0.50	-0.01
Fabricated metal	0.53	1.90	0.61
Metal mining	0.57	0.11	0.07
Oil and gas extraction	0.63	2.32	-0.23
Services	0.64	21.61	-0.56
Lumber and wood	0.77	0.75	0.18

Notes: We use the full set of KLEM industries provided in Jorgenson (2008). The industrial share, s_i, refers to the time-series average share of industry i in total value added, that is, $s_i = \sum_t \frac{s_{it}}{T}$ where for each period $s_{it} = \frac{y_{it}}{y_t}$ and y_t refers to the total value added computed as the sum of all industries value added, that is, $y_t = \sum_i y_{it}$. To compute the cyclical correlation between the industrial share, s_{it}, and aggregate output, y_t, we first log and HP-filter these two series.

Figure C-1: Industry-Specific Elasticity of Substitution, σ_i, vs. Cyclical Correlation of Industrial Shares and Aggregate Output, KLEM 1960-2005



Notes: See footnote of Table C-1.

D The Propagation Power of σ_t: A Theoretical Description

To see this, recall that σ appears in our model only through its effects on the production function. This way, we can study the potential effects of σ on our model allocations by analyzing its impact on our technology. Take our SR-LR production function defined in its general form as in (2), which we can compactly rewrite (dropping time subscripts) as

$$F(\mathbf{x}, \sigma) = f^{SR}(\widehat{\mathbf{x}}, \sigma)\, f^{LR}(\mathbf{x}^*, 1), \tag{11}$$

where $\mathbf{x}$ is an m-dimensional vector with elements $i = \{1, m\}$ including all the factor inputs of production and labor-augmenting technical change. We denote this vector evaluated at the steady state as $\mathbf{x}^*$, and $\widehat{\mathbf{x}}$ is the result of the Hadamard product $\mathbf{x} \circ .\mathbf{x}^*$ where $.\mathbf{x}^*$ inverts all the elements in vector $\mathbf{x}^*$. This way, each element in $\widehat{\mathbf{x}}$ is a ratio between an actual level and its respective steady-state value.[12] Therefore, $\widehat{\mathbf{x}}^* = 1$ at steady state. As we proposed in Section 2, our production function has two components: $f^{SR}(\widehat{\mathbf{x}}, \sigma)$ is the short-run component of the production function that is potentially shaped by σ over time, and $f^{LR}(\mathbf{x}^*, 1)$ is the long-run component of the production function with input factors evaluated at the steady state and with an elasticity of substitution equal to one.

Next, we take the first-order approximation (Taylor polynomial) of our production function around its steady state.[13] First, note that f^{LR} has no role (it is a constant) when the production function is linearized, and hence we can normalize it to one and drop it from the current analysis without loss of generality. This way, we focus only on the short-run component of the production hereafter. We also drop the superscript SR. This way, the first-order approximation of our scalar-valued function (D) around steady state is

$$f(\widehat{\mathbf{x}}, \sigma) \approx P_1(\widehat{\mathbf{x}}, \sigma) = f|_* + \sum_{i}^{m} f_i|_* (\widehat{x}_i - \widehat{x}_i^*) + f_\sigma|_* (\sigma - \sigma^*), \tag{12}$$

where $f|_* = f(\widehat{\mathbf{x}}^*, \sigma^*)$ is the short-run component of the production function (i.e., short-run output) evaluated at steady state; $(\widehat{\mathbf{x}}^*, \sigma^*) = (1, \sigma^*)$, $f_i|_* = \left.\frac{\partial f}{\partial \widehat{x}_i}\right|_*$ denotes the first partial derivative of f with respect to each element $\widehat{x}_i \in \widehat{\mathbf{x}}$ evaluated at steady state; and the last term of the gradient of f is $f_\sigma|_* = \left.\frac{\partial f}{\partial \sigma}\right|_*$, which is the first partial derivative of f with respect to σ. Under a first order approximation of f, this last term is the only one that captures *ceteris paribus* the potential effects of shocks to σ on the short-run equilibrium.

However, note that with our specific NCES short-run technology,

$$f(\widehat{\mathbf{x}}, \sigma) = g(\widehat{\mathbf{x}}, \varphi(\sigma))^{\varphi(\sigma)},$$

where

$$g(\widehat{\mathbf{x}}, \varphi(\sigma)) \equiv ((1-\alpha)(\widehat{u}\widehat{k})^{1/\varphi(\sigma)} + \alpha(\widehat{a}\widehat{e}\widehat{h})^{1/\varphi(\sigma)}) \tag{13}$$

with $\varphi(\sigma) \equiv \frac{\sigma}{\sigma - 1}$, and the first partial derivative of f with respect to σ is[14]

$$\frac{\partial f}{\partial \sigma} = f\, \frac{\partial \varphi}{\partial \sigma} \left(\log(g) + \varphi(\sigma) g^{-1} \frac{\partial g}{\partial \varphi} \right).$$

This way, if this partial derivative (i.e., the effect of σ) is computed around the steady sate, then we find from equation (13) that $g(\widehat{\mathbf{x}}^*, \sigma) = g(1, \varphi(\sigma^*)) = 1$ and $\frac{\partial g}{\partial \varphi}(1, \varphi(\sigma)) = 0$ and, hence, $\left.\frac{\partial f}{\partial \sigma}\right|_* = 0$. This conveys the

[12] That is, as we defined in equation (1), the log of each variable in vector $\widehat{\mathbf{x}}$ represents the log-deviation from steady state of that respective variable.

[13] We explicitly drop the perturbation term for expositional simplicity when we expand the production function; yet, one may consider that the perturbation term is implicitly incorporated in the m-dimensional vector $\mathbf{x}$.

[14] Note that we use the fact that $\frac{\partial f}{\partial \sigma} = f \cdot \frac{\partial \log f}{\partial \sigma}$.

message that with a first-order approximation of our model, shocks to σ bear absolutely no impact on f (and hence they do not have an effect on y_t, w_t, r_t or on any other endogenous variables either). We summarize this result in the following Lemma.

Lemma 1. *In a first-order approximation around steady state, shocks to the elasticity of substitution have no impact on f (i.e., on equilibrium allocations).*

It is clear that the first-order approximation leaves no scope for effects of fluctuations in σ_t on equilibrium allocations. However, we know from our discussion in Section 2.3.1 that our model is potentially highly nonlinear and, hence, a first-order approximation is not an appropriate solution method as it can generate a high degree of inaccuracy on our computed decision rules. As we discussed, our choice is to resolve this nonlinearity issue by using a third-order perturbation method. Next, we look at the formulae, in extensive form, of the third-order approximation that we use in our computations. This provides new insights about the channels through which fluctuations in σ can actually have an impact on f.

The third-order approximation of f is

$$
\begin{aligned}
P_3(\widehat{\mathbf{x}},\sigma) = f|_* &+ \sum_{i}^{m} f_{\widehat{x}_i}|_* (\widehat{x}_i - \widehat{x}_i^*) + f_\sigma|_* (\sigma - \sigma^*) \\
&+ \frac{1}{2!}\left[\sum_{i=1}^{m}\sum_{j=1}^{m} f_{x_j x_i}\big|_* (x_i - x_i^*)(x_j - x_j^*) + 2\sum_{i=1}^{m} f_{x_i\sigma}|_* (x_i - x_i^*)(\sigma - \sigma^*) + f_{\sigma\sigma}|_* (\sigma - \sigma^*)^2\right] \\
&+ \frac{1}{3!}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} f_{x_k x_j x_i}\big|_* (x_i - x_i^*)(x_j - x_j^*)(x_k - x_k^*)\right] \\
&+ \frac{1}{3!}\left[3\sum_{j=1}^{m}\sum_{i=1}^{m} f_{\sigma x_j x_i}\big|_* (x_i - x_i^*)(x_j - x_j^*)(\sigma - \sigma^*)\right] \\
&+ \frac{1}{3!}\left[3\sum_{i=1}^{m} f_{\sigma\sigma x_i}|_* (x_i - x_i^*)(\sigma - \sigma^*)^2\right] \\
&+ \frac{1}{3!}\left[f_{\sigma\sigma\sigma}|_* (\sigma - \sigma^*)^3\right] \qquad (14)
\end{aligned}
$$

where the first line in (14) is the first-order approximation of f, $P_1(\widehat{\mathbf{x}},\sigma)$. The first and second lines in (14) represent the second-order approximation of f, $P_2(\widehat{\mathbf{x}},\sigma)$, where $f_{x_j x_i} = \frac{\partial^2 f}{\partial x_i \partial x_j}$ for all pairs (i,j) are the elements in the submatrix of the Hessian of f that includes all but its last column and row, the term $f_{x_i\sigma}$ captures the cross partial derivatives between $\widehat{\mathbf{x}}$ and σ (i.e., the off-diagonal elements in the last row and column of the Hessian)[15] and $f_{\sigma\sigma}$ is the second-order partial derivative of f with respect to σ (i.e., the diagonal bottom-right element of the Hessian). Finally, the last four lines in (14) capture the terms added by the third-order approximation where $f_{x_k x_j x_i} = \frac{\partial^3 f}{\partial x_i \partial x_j \partial x_k}$ for all triplets (i,j,k), $f_{\sigma x_j x_i} = \frac{\partial^3 f}{\partial x_i \partial x_j \partial \sigma}$ for all pairs (i,j), $f_{\sigma\sigma x_i} = \frac{\partial^3 f}{\partial x_i \partial \sigma^2}$ $\forall i$, and $f_{\sigma\sigma\sigma} = \frac{\partial^3 f}{\partial \sigma^3}$ is the third-order partial derivative of f with respect to σ.[16] Our third-order approximation sheds light on the effects of σ.

Theorem 1. *In a nth-order approximation around steady state:*

(a) *Shocks to the elasticity of substitution, σ, have no impact on f (i.e., on equilibrium allocations) through*

[15] Note that in (14) we have used the symmetry of the Hessian, $f_{\sigma x_i} = f_{x_i\sigma}$ (Clariaut's theorem).

[16] Note that in (14) we have used the fact that we can exchange the order in which we take the cross partial derivatives, $f_{\sigma x x} = f_{x\sigma x} = f_{x x\sigma}$ and $f_{\sigma\sigma x} = f_{\sigma x\sigma} = f_{x\sigma\sigma}$, due to the continuity of each of these cross partial derivatives of f (an extension of Clariaut's theorem)

the partial derivatives $\frac{\partial^n f}{\partial \sigma^n}\Big|_*$ *of any order* $n > 0$*. Hence,* σ*-shocks are not a source of fluctuations per se.*

(b) *If* $n > 1$ *then shocks to* σ *propagate the effects of other sources of fluctuations on* f *(i.e., on equilibrium allocations) through the higher-order cross partial derivatives of* f *between* σ *and* $\widehat{\mathbf{x}}$*.*

Proof. (a) From Lemma 1 we know that the partial derivative of f with respect to σ evaluated at steady state is zero. This implies that second- and third-order partial derivatives of f with respect to σ are also equal to zero when evaluated at the steady state, $\frac{\partial f}{\partial \sigma}\Big|_* = \frac{\partial^2 f}{\partial \sigma^2}\Big|_* = \frac{\partial^3 f}{\partial \sigma^3}\Big|_* = 0$. This result easily extends to higher-order derivatives. That is, a model that is at steady state and faces no other shock than a shock to the elasticity of substitution will not move its allocations away from steady state because $\frac{\partial^n f}{\partial \sigma^n}\Big|_*$ is zero $\forall n > 0$. This means that shocks to σ are not a source of fluctuations *per se*.

(b) It is straightforward to see from (14) that fluctuations in σ can have an effect on equilibrium allocations generated by the second cross partial derivatives between σ and $\widehat{\mathbf{x}}$ in (14),

$$2\sum_{i=1}^{m} f_{x_i\sigma}|_* \, (x_i - x_i^*)(\sigma - \sigma^*), \tag{15}$$

and the third cross partial derivatives between σ and $\widehat{\mathbf{x}}$ in (14), that is,

$$3\sum_{j=1}^{m}\sum_{i=1}^{m} f_{\sigma x_j x_i}|_* \, (x_i - x_i^*)(x_j - x_j^*)(\sigma - \sigma^*), \text{ and } 3\sum_{i=1}^{m} f_{\sigma\sigma x_i}|_* \, (x_i - x_i^*)(\sigma - \sigma^*)^2. \tag{16}$$

However, by Clariaut's theorem we can exchange the order of cross partial derivatives. This means that to show that σ has an effect through the second- or any higher-order cross partial derivatives, given its existence, it is sufficient to show that the second-order cross partial derivatives between σ and any factor input $\widehat{x}_i \in \widehat{\mathbf{x}}$ is non-zero when evaluated at steady state. We can write such a second cross partial derivative as

$$\begin{aligned}\frac{\partial^2 f}{\partial \sigma \partial x_1} =& \frac{\partial f}{\partial x_1}\frac{\partial \varphi}{\partial \sigma}\left(\log(g) + \varphi(\sigma) g^{-1}\frac{\partial g}{\partial \varphi}\right) + \\ & f\,\frac{\partial \varphi}{\partial \sigma}\left(g^{-1}\frac{\partial g}{\partial x_1} - \varphi(\sigma) g^{-2}\frac{\partial g}{\partial x_1}\frac{\partial g}{\partial \varphi} + \varphi(\sigma) g^{-1}\frac{\partial^2 g}{\partial \varphi \partial x_1}\right).\end{aligned} \tag{17}$$

When this derivative is evaluated at steady state $\widehat{\mathbf{x}}^* = \mathbf{1}$, then

$$\frac{\partial^2 f}{\partial \sigma \partial x_1}\Big|_* = \frac{\partial \varphi}{\partial \sigma}\Big|_*\left(\frac{\partial g}{\partial x_1}\Big|_* + \varphi(\sigma)\,\frac{\partial^2 g}{\partial \varphi \partial x_1}\Big|_{\mathbf{x}^*,\sigma^*}\right) \neq 0. \tag{18}$$

This implies that fluctuations in σ have a potential propagation effect on the equilibrium allocations when other shocks in the model are already operative (i.e., nonzero). That is, σ propagates existing sources of fluctuations. □

Summing up, not only do we need higher-order perturbation methods to accurately solve for decision rules that are potentially nonlinear due to fluctuations of σ, but also we have showed that if we solve our model with a first-order approximation, then, by construction, fluctuations of σ_t will have absolutely no effect on our allocations because the cross partial derivatives between σ and $\widehat{\mathbf{x}}$ that capture the effect σ are only present in second-order or higher-order approximations. That is, a first-order approximation entirely underestimates the potential role of σ for the business cycle. Only through higher-than-first-order perturbation (or other global methods for that matter) can we accurately solve for the decision rules and also capture the actual propagation power of σ when other shocks are present.

Higher Order Perturbation Methods The nonconstancy of the elasticity of substitution σ_t implies that nonlinear techniques are required to solve our model. To deal with this issue, we choose a third-order perturbation

method. This perturbation method can handle a large set of state variables in the model and guarantees a certain degree of accuracy despite its feature of 'local' approximation (see Aruoba et al. (2006)). Our model contains five shocks, and four of them exhibit AR(2) dynamics. This implies a total of ten state variables, nine of which come from the exogenous shocks and one from the capital stock, the only endogenous state variable. Hence, our main computational concern lies with the "curse of dimensionality." Standard solution methods for nonlinear models are perturbation methods, projection methods, and value function iteration (VFI). However, as is well known, projection methods and VFI both suffer from the "curse of dimensionality" (see a comprehensive discussion in Aruoba et al. (2006)). For example, in the context of our model, a projection method with second-order Chebyshev polynomials would involve 3^{10} terms to be estimated to approximate a single policy function, which is computationally very intensive and time costly.[17] Further, even though VFI is a reliable and an easy-to-implement global solution, the VFI computation with ten state variables for a reasonable number of grid points in the state space is a Dantesque task independent of the tensor product that we choose.

Since we have to solve this model many times for the purpose of structural estimation (our solution method is embedded into the simulated method of moments algorithm; see Section F below), we find that a third-order perturbation method is well suited for our purposes, displaying a good balance between speed and accuracy.[18]

[17] Malin et al. (2007) address this issue with the use of Smolyak polynomials in collocation.

[18] Aruoba et al. (2006) provide a comprehensive analysis of these and additional methods with a standard RBC model of three state variables, capital, productivity, and investment-specific shocks. They assert that higher-order perturbation methods (at least second-order) display a superior performance over alternative methods in terms of accuracy, speed and coding burden.

E Model Equilibrium Conditions and Stationarization

Here we explicitly pose the equilibrium conditions that yield our optimal allocations. We also discuss the stationarization of our model.

E.1 Planner's Problem

In per capita terms (dividing by N_t) the social planner's problem is:

$$\max_{\{c_t,h_t,e_t,u_t\}_{t=0}^{\infty}} E_0 \sum \beta^t \left\{ \log c_t - \kappa_1 \frac{h_t^{1+\frac{1}{\nu_1}}}{1+\frac{1}{\nu_1}} e_t + \kappa_2 \frac{(b_t(1-e_t))^{1+\frac{1}{\nu_2}}}{1+\frac{1}{\nu_2}} \right\} \tag{19}$$

s.t.

$$c_t + i_t + g_t = y_t \tag{20}$$

$$v_t i_t = k_{t+1}(1+\lambda_n) - \left(1 - \left(\delta_0 + \delta_1 \left(u_t^{1+\frac{1}{\zeta}} - 1\right)\right)\right) k_t \tag{21}$$

$$y_t = \begin{cases} \left((1-\alpha) \left(\frac{u_t k_t}{u_t^* k_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} + \alpha \left(\frac{a_t e_t h_t}{a_t^* e_t^* h_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \right)^{\frac{\sigma_t}{\sigma_t - 1}} (u_t^* k_t^*)^{1-\theta} (a_t^* e_t^* h_t^*)^{\theta}, & \text{if} \quad \widehat{s}_t \neq 1 \\ (u_t^* k_t^*)^{1-\theta} (a_t^* e_t^* h_t^*)^{\theta}, & \text{if} \quad \widehat{s}_t = 1 \end{cases} \tag{22}$$

and exogenous processes $s_t = \{a_t, v_t, g_t, b_t, \sigma_t\}$ with given k_0, a_0, v_0, g_0, b_0 and σ_0.

E.2 First Order Conditions (FOC)

$$\text{FOC(h):} \quad \frac{1}{c_t} \frac{\partial y_t}{\partial h_t} - \kappa_1 h_t^{\frac{1}{\nu_1}} e_t = 0 \tag{23}$$

$$\text{FOC(e):} \quad \frac{1}{c_t} \frac{\partial y_t}{\partial e_t} - \kappa_1 \frac{h_t^{1+\frac{1}{\nu_1}}}{1+\frac{1}{\nu_1}} + \kappa_2 b_t^{1+\frac{1}{\nu_2}} (1-e_t)^{\frac{1}{\nu_2}} (-1) = 0 \tag{24}$$

$$\text{FOC(u):} \quad \frac{1}{c_t} \left[\frac{\partial y_t}{\partial u_t} - \frac{\partial i_t}{\partial u_t} \right] = 0 \tag{25}$$

$$\text{FOC(k')}: \quad -\beta^t \frac{1}{c_t v_t} (1+\lambda_n) + \beta^{t+1} E \frac{1}{c_{t+1}} \left[\frac{\partial y_{t+1}}{\partial k_{t+1}} - \frac{\partial i_{t+1}}{\partial k_{t+1}} \right] = 0 \tag{26}$$

E.2.1 Marginal Products

$$\begin{aligned} MP_{eh} = \frac{\partial y_t}{\partial e_t h_t} &= \frac{\partial f_t^{SR}}{\partial e_t h_t} f^{LR} \\ &= \frac{\sigma_t}{\sigma_t - 1} \left(\frac{y_t}{y_t^*}\right)^{\frac{1}{\sigma_t}} \alpha \frac{\sigma_t - 1}{\sigma_t} \left(\frac{a_t}{a_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{e_t h_t}{e_t^* h_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{1}{e_t^* h_t^*} f^{LR} \\ &= \alpha \left(\frac{a_t}{a_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{e_t h_t / e_t^* h_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{y_t^*}{e_t^* h_t^*} = w_t \end{aligned} \tag{27}$$

$$MP_h = \frac{\partial y_t}{\partial h_t} = \frac{\partial f_t^{SR}}{\partial h_t} f^{LR} = \alpha \left(\frac{a_t e_t}{a_t^* e_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{h_t / h_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{y_t^*}{h_t^*} = w_t e_t \tag{28}$$

$$MP_e = \frac{\partial y_t}{\partial e_t} = \frac{\partial f_t^{SR}}{\partial e_t} f^{LR} = \alpha \left(\frac{a_t h_t}{a_t^* h_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{e_t / e_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{y_t^*}{e_t^*} = w_t h_t \tag{29}$$

$$MP_k = \frac{\partial y_t}{\partial k_t} = \frac{\partial f_t^{SR}}{\partial k_t} f^{LR} = (1-\alpha) \left(\frac{u_t}{u_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{k_t / k_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{y_t^*}{k_t^*} = r_t \tag{30}$$

$$MP_u = \frac{\partial y_t}{\partial u_t} = \frac{\partial f_t^{SR}}{\partial u_t} f^{LR} = (1-\alpha) \left(\frac{k_t}{k_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} \left(\frac{u_t / u_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}} \frac{y_t^*}{u_t^*} = r_t \frac{k_t}{u_t} \tag{31}$$

and

$$\frac{\partial i_t}{\partial u_t} = \delta_1 \left(1 + \frac{1}{\zeta}\right) u_t^{\frac{1}{\zeta}} \frac{k_t}{v_t} \tag{32}$$

$$\frac{\partial i_t}{\partial k_t} = \left(1 - \left(\delta_0 + \delta_1 \left(u_t^{1+\frac{1}{\zeta}} - 1\right)\right)\right) \frac{1}{v_t} \tag{33}$$

E.2.2 Plugging marginal products into FOCS

FOC w.r.t h_t

$$\begin{aligned}
&\frac{1}{c_t} \frac{\partial y_t}{\partial h_t} = \kappa_1 h_t^{\frac{1}{\nu_1}} e_t \\
&\frac{1}{c_t} w_t e_t = \kappa_1 h_t^{\frac{1}{\nu_1}} e_t \\
&\frac{1}{c_t} w_t = \kappa_1 h_t^{\frac{1}{\nu_1}}
\end{aligned} \tag{34}$$

FOC w.r.t e_t

$$\begin{aligned}
&\frac{1}{c_t} \frac{\partial y_t}{\partial e_t} - \kappa_1 \frac{h_t^{1+\frac{1}{\nu_1}}}{1+\frac{1}{\nu_1}} = \kappa_2 b_t^{1+\frac{1}{\nu_2}} (1-e_t)^{\frac{1}{\nu_2}} \\
&\frac{1}{c_t} w_t h_t - \kappa_1 \frac{h_t^{1+\frac{1}{\nu_1}}}{1+\frac{1}{\nu_1}} = \kappa_2 b_t^{1+\frac{1}{\nu_2}} (1-e_t)^{\frac{1}{\nu_2}} \\
&\left(1 - \frac{1}{1+\frac{1}{\nu_1}}\right) \frac{1}{c_t} w_t h_t = \kappa_2 b_t^{1+\frac{1}{\nu_2}} (1-e_t)^{\frac{1}{\nu_2}} \\
&\frac{1}{1+\nu_1} \frac{1}{c_t} w_t h_t = \kappa_2 b_t^{1+\frac{1}{\nu_2}} (1-e_t)^{\frac{1}{\nu_2}}
\end{aligned} \tag{35}$$

where we have used a formulation of the previous FOC(h): $\frac{1}{c_t} w_t h_t = \kappa_1 h_t^{1+\frac{1}{\nu_1}}$.

FOC w.r.t u_t

$$\begin{aligned}
&\frac{\partial y_t}{\partial u_t} = \frac{\partial i_t}{\partial u_t} \\
&r_t \frac{k_t}{u_t} = \delta_1 \left(1 + \frac{1}{\zeta}\right) u_t^{\frac{1}{\zeta}} \frac{k_t}{v_t} \\
&r_t v_t = \delta_1 \left(1 + \frac{1}{\zeta}\right) u_t^{1+\frac{1}{\zeta}}
\end{aligned} \tag{36}$$

FOC w.r.t k_{t+1}

$$\frac{1}{c_t v_t}(1+\lambda_n) = \beta E \frac{1}{c_{t+1}}\left[\frac{\partial y_{t+1}}{\partial k_{t+1}} - \frac{\partial i_{t+1}}{\partial k_{t+1}}\right]$$
$$\frac{1}{c_t v_t}(1+\lambda_n) = \beta E \frac{1}{c_{t+1}}\left[r_{t+1} + \left(1 - \left(\delta_0 + \delta_1\left(u_{t+1}^{1+\frac{1}{\zeta}} - 1\right)\right)\right)\frac{1}{v_{t+1}}\right] \tag{37}$$
$$(1+\lambda_n) = \beta E \frac{c_t}{c_{t+1}}\frac{v_t}{v_{t+1}}\left[r_{t+1}v_{t+1} + 1 - \left(\delta_0 + \delta_1\left(u_{t+1}^{1+\frac{1}{\zeta}} - 1\right)\right)\right]$$

E.3 Full Set of Equilibrium Conditions

$$\frac{1}{c_t} w_t = \kappa_1 h_t^{\frac{1}{\nu_1}} \tag{38}$$
$$\frac{1}{1+\nu_1}\frac{1}{c_t} w_t h_t = \kappa_2 b_t^{1+\frac{1}{\nu_2}}(1-e_t)^{\frac{1}{\nu_2}} \tag{39}$$
$$r_t v_t = \delta_1\left(1 + \frac{1}{\zeta}\right)u_t^{1+\frac{1}{\zeta}} \tag{40}$$
$$(1+\lambda_n) = \beta E \frac{c_t}{c_{t+1}}\frac{v_t}{v_{t+1}}\left[r_{t+1}v_{t+1} + 1 - \left(\delta_0 + \delta_1\left(u_{t+1}^{1+\frac{1}{\zeta}} - 1\right)\right)\right] \tag{41}$$
$$w_t = \alpha\left(\frac{a_t}{a_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}}\left(\frac{e_t h_t / e_t^* h_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}}\frac{y_t^*}{e_t^* h_t^*} \tag{42}$$
$$r_t = (1-\alpha)\left(\frac{u_t}{u_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}}\left(\frac{k_t / k_t^*}{y_t / y_t^*}\right)^{-\frac{1}{\sigma_t}}\frac{y_t^*}{k_t^*} \tag{43}$$
$$c_t + i_t + g_t = y_t \tag{44}$$
$$v_t i_t = k_{t+1}(1+\lambda_n) - \left(1 - \left(\delta_0 + \delta_1\left(u_t^{1+\frac{1}{\zeta}} - 1\right)\right)\right)k_t \tag{45}$$
$$y_t = \begin{cases} \left((1-\alpha)\left(\frac{u_t k_t}{u_t^* k_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} + \alpha\left(\frac{a_t e_t h_t}{a_t^* e_t^* h_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}}\right)^{\frac{\sigma_t}{\sigma_t - 1}}(u_t^* k_t^*)^{1-\theta}(a_t^* e_t^* h_t^*)^{\theta}, & \text{if} \quad \widehat{\mathcal{Z}}_t \neq 1 \\ (u_t^* k_t^*)^{1-\theta}(a_t^* e_t^* h_t^*)^{\theta}, & \text{if} \quad \widehat{\mathcal{Z}}_t = 1 \end{cases} \tag{46}$$

Or, more conveniently,

$$y_t = \left((1-\alpha)\left(\frac{u_t k_t}{u_t^* k_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}} + \alpha\left(\frac{a_t e_t h_t}{a_t^* e_t^* h_t^*}\right)^{\frac{\sigma_t - 1}{\sigma_t}}\right)^{\frac{\sigma_t}{\sigma_t - 1}}(u_t^* k_t^*)^{1-\theta}(a_t^* e_t^* h_t^*)^{\theta} \tag{47}$$
$$y_t^* = (u_t^* k_t^*)^{1-\theta}(a_t^* e_t^* h_t^*)^{\theta} \tag{48}$$

and exogenous processes $\{a_t, v_t, g_t, b_t, \sigma_t\}$ with given k_0, a_0, v_0, g_0, b_0 and σ_0.

E.4 Stationarizing the model

Along the BGP, y, c, i, w, g are growing at a rate λ_y, k is growing at a rate λ_k, a is growing at a rate λ_a, and v is growing at a rate λ_v. Then, $(1+\lambda_k) = (1+\lambda_y)(1+\lambda_v)$ must hold from equation (43) and $(1+\lambda_y) = (1+\lambda_v)^{\frac{1-\theta}{\theta}}(1+\lambda_a)$ from equation (47); rv is stationary (see equation (40)). The set of variables e, h, u, b is

also stationary. The equilibrium conditions in its stationary form are:[19]

$$\frac{1}{\widetilde{c}_t}\widetilde{w}_t = \kappa_1 \widetilde{h}_t^{\frac{1}{\nu_1}} \tag{49}$$

$$\frac{1}{1+\nu_1}\frac{1}{\widetilde{c}_t}\widetilde{w}_t\widetilde{h}_t = \kappa_2 \widetilde{b}_t^{1+\frac{1}{\nu_2}}(1-\widetilde{e}_t)^{\frac{1}{\nu_2}} \tag{50}$$

$$\widetilde{r}_t\widetilde{v}_t = \delta_1\left(1+\frac{1}{\zeta}\right)\widetilde{u}_t^{1+\frac{1}{\zeta}} \tag{51}$$

$$(1+\lambda_n)(1+\lambda_k) = \beta E\frac{\widetilde{c}_t}{\widetilde{c}_{t+1}}\frac{\widetilde{v}_t}{\widetilde{v}_{t+1}}\left[\widetilde{r}_{t+1}\widetilde{v}_{t+1}+1-\left(\delta_0+\delta_1\left(\widetilde{u}_{t+1}^{1+\frac{1}{\zeta}}-1\right)\right)\right] \tag{52}$$

$$\widetilde{w}_t = \alpha\left(\frac{\widetilde{a}_t}{a_0}\right)^{\frac{\sigma_t-1}{\sigma_t}}\left(\frac{\widetilde{e}_t\widetilde{h}_t/e_0h_0}{\widetilde{y}_t/y_0}\right)^{-\frac{1}{\sigma_t}}\frac{y_0}{e_0h_0} \tag{53}$$

$$\widetilde{r}_t = (1-\alpha)\left(\frac{\widetilde{u}_t}{u_0}\right)^{\frac{\sigma_t-1}{\sigma_t}}\left(\frac{\widetilde{k}_t/k_0}{\widetilde{y}_t/y_0}\right)^{-\frac{1}{\sigma_t}}\frac{y_0}{k_0} \tag{54}$$

$$\widetilde{c}_t + \widetilde{i}_t + \widetilde{g}_t = \widetilde{y}_t \tag{55}$$

$$\widetilde{v}_t\widetilde{i}_t = \widetilde{k}_{t+1}(1+\lambda_n)(1+\lambda_k) - \left(1-\left(\delta_0+\delta_1\left(\widetilde{u}_t^{1+\frac{1}{\zeta}}-1\right)\right)\right)\widetilde{k}_t \tag{56}$$

$$\widetilde{y}_t = \left((1-\alpha)\left(\frac{\widetilde{u}_t\widetilde{k}_t}{u_0k_0}\right)^{\frac{\sigma_t-1}{\sigma_t}} + \alpha\left(\frac{\widetilde{a}_t\widetilde{e}_t\widetilde{h}_t}{a_0e_0h_0}\right)^{\frac{\sigma_t-1}{\sigma_t}}\right)^{\frac{\sigma_t}{\sigma_t-1}}(u_0k_0)^{1-\theta}(a_0e_0h_0)^{\theta} \tag{57}$$

$$y_0 = (u_0k_0)^{1-\theta}(a_0e_0h_0)^{\theta} \tag{58}$$

and exogenous processes $\{a_t, v_t, g_t, b_t, \sigma_t\}$ with given k_0, a_0, v_0, g_0, b_0 and σ_0.

[19] Denote $\widetilde{x}_t = \frac{x_t}{(1+\lambda_x)^t} = x_0\hat{x}_t$.

F A Simulation-Based Estimation

We use the full structure of our model, via simulation, to estimate the remaining set of parameters. We pose a set of targeted moments that we want our model-generated data to match. These targets are (1) a set of standard unconditional business cycle moments of the labor market (seven moments) and (2) the IRF of productivity and labor share in response to productivity shocks (sixteen moments) as specified in Section 1.

The set of unconditional business cycle moments (from logged and HP-filtered variables) consists of the variance of output per capita $var(y)$ and labor share $var(ls)$; the correlation between wages and output per capita, $\rho(w,y)$; the correlation between labor productivity and output per capita, $\rho(lp,y)$; the correlation between labor share and output per capita, $\rho(ls,y)$; and the correlation between employment per capita and average hours per worker, $\rho(e,h)$. We also target the long-run (steady-state) average of employment per capita, e_0. In addition, we target the data IRFs computed by the estimated trivariate AR process of productivity, labor share and output per capita. This trivariate is specified as discussed in Section B.2; that is, each polynomial $\Psi(L)$ associated with persistence parameters is assumed to have two lags, and the orthogonalized variance-covariance matrix $\Omega_{z,ls,y}$ has five non-zero elements, the three elements on the diagonal and the two contemporaneous effects of productivity innovations on labor share $\omega_{z,ls}$ and output $\omega_{z,y}$. In an effort to reduce the number of targeting moments, rather than incorporating $\Psi(L)$ and $\Omega_{z,ls,y}$ as moments, we choose to target a set of dynamic multipliers of $\Theta_{z,z} = \{\theta_{z,z}^{(s_1)}, \theta_{z,z}^{(s_2)}, \ldots, \theta_{z,z}^{(s_n)}\}$ and $\Theta_{z,ls} = \{\theta_{z,ls}^{(s_1)}, \theta_{z,ls}^{(s_2)}, \ldots, \theta_{z,ls}^{(s_n)}\}$ where $\theta_{u,x}^{s} = \frac{\partial x_{t+s}}{\partial u_t}$ is the response of x at period $t+s$ to a shock u at t. Note that we do not target the response of output to productivity (i.e., $\Theta_{z,y}$). Instead, we will use the performance of output behavior to discuss the external validation of our model. The periods of dynamic multipliers that we choose to match are associated with the following eight periods, $\{s_1 = 1, s_2 = 2, s_3 = 5, s_4 = 10, s_5 = 20, s_6 = 30, s_7 = 40, s_8 = 50\}$, balancing the recovery of initial responses and potential long-lasting impacts. Our estimation strategy is summarized by the following algorithm.

Algorithm 1 *A simulation-based estimation:*

STEP 1. *Define the full set of targeted data moments as the following vector (23 moments),*

$$\phi^d = \{var(y),\ var(ls),\ \rho(w,y),\ \rho(lp,y),\ \rho(ls,y),\ \rho(e,h),\ e_0,\ \Theta_{z,z},\ \Theta_{z,ls}\}\,.$$

STEP 2. *Guess values for the set of parameters that we want to estimate (19 parameters) :*

$$\gamma = \{\sigma,\ \Psi_{a,a}(L),\ \Psi_{\sigma,\sigma}(L),\ \Psi_{a,\sigma}(L),\ \Omega_{a,\sigma},\ \Psi_{b,b}(L),\ \upsilon_b,\ \Psi_{v,v}(L),\ \upsilon_v,\ \zeta,\ \nu_2,\ \kappa_2\}, \tag{59}$$

where we specify two lags in the operator L, and allow a and σ to be correlated. Specifically,

$$a_t = \Psi_{a,a}(L)a_t + \epsilon_{a,t} \tag{60}$$
$$\sigma_t = \Psi_{\sigma,\sigma}(L)\sigma_t + \Psi_{a,\sigma}(L)a_t + \epsilon_{\sigma,t} \tag{61}$$

with

$$\boldsymbol{\epsilon}_t = \begin{pmatrix} \epsilon_{a,t} \\ \epsilon_{\sigma,t} \end{pmatrix} \sim N(0, \Sigma_{a,\sigma}), \tag{62}$$

which we factorize a la Cholesky to yield,

$$\boldsymbol{u}_t = \Omega_{a,\sigma}\boldsymbol{\epsilon}_t \quad \text{and} \quad \Omega_{a,\sigma} = \begin{pmatrix} \upsilon_a & 0 \\ \upsilon_{a,\sigma} & \upsilon_\sigma \end{pmatrix}, \tag{63}$$

where $\boldsymbol{u}_t$ has unit variance, and the identification assumption to pin down $\Omega_{a,\sigma}$ is that innovations to productivity affect the elasticity of substitution between capital and labor (i.e., $\upsilon_{a,\sigma} \neq 0$) but not the other way around (i.e., $\upsilon_{\sigma,a} = 0$); that is, $\Omega_{a,\sigma}$ is a lower triangular matrix.

The home-productivity shocks follow

$$b_t = \Psi_{b,b}(L)b_t + \epsilon_{b,t} \quad \text{and} \quad \epsilon_b \sim N(0, \upsilon_b).$$

The investment-specific technical change shocks follow

$$v_t = \Psi_{v,v}(L)v_t + \epsilon_{v,t} \quad \text{and} \quad \epsilon_v \sim N(0, \upsilon_v).$$

STEP 3. *Given γ, solve and simulate the model economy.*[20]

STEP 4. (a) *Use model-simulated data to compute the model counterpart of the productivity residual z constructed from the data in Appendix A.6. To do so we use a model-generated series of capital $\widetilde{k}_t$ that has the same properties as the series of capital used in the data computation of z_t. Specifically, we use a model-generated series of capital that assumes a constant depreciation rate; that is, $\widetilde{k}_t = k_t|_{u^*,v^*}$ with $u_t = u^* = 1$ and $v_t = v^* = 1$. That is, we recover $\widetilde{k}_t$ from*

$$i_t = \widetilde{k}_{t+1} - (1 - \delta_0)\widetilde{k}_t, \tag{64}$$

with model investment i_t, steady-state (initial) capital k^*, stationary depreciation rate $\delta(u^*) = \delta_0$, $\widetilde{k}_0 = k^*$ and investment-specific technical change v_t. Finally, we use $\widetilde{k}_t$ together with the model series for output y_t, labor eh_t and labor share ls_t to find

$$\lambda_{z_t} = \lambda_{y_t} - (1 - ls_t)\lambda_{\widetilde{k}_t} - ls_t\lambda_{eh_t}, \tag{65}$$

where $\lambda_{x_t} = \frac{x_t - x_{t-1}}{x_{t-1}}$ is a growth rate of a variable x. Normalizing z_0=1, we recover the entire series z_t.

(b) *Use the productivity residual z_t from the previous step (a) and the model series for labor share ls_t and output y_t to estimate the trivariate AR process (8)-(10) in Appendix B.2 and its associated IRFs.*

STEP 5. *Define the set of model-generated moments of interest as*

$$\phi^m(\gamma) = \{var(y),\ var(ls),\ \rho(w,y),\ \rho(lp,y),\ \rho(ls,y),\ \rho(e,h),\ e_0,\ \Theta_{z,z},\ \Theta_{z,ls}\}.$$

STEP 6. *Iterate our estimated values γ until we minimize the distance between the model moments $\phi^m(\gamma)$ and the data moments ϕ^d. That is, our estimator γ solves*

$$J = \min_{\gamma} \left[\phi^d - \phi^m(\gamma)\right]' W^{-1} \left[\phi^d - \phi^m(\gamma)\right]$$

where we use an estimated variance-covariance matrix of ϕ^d (Ω^d) adjusted by the number of simulations (N) and samples (T) as an optimal weighting $W = (1 - T/N)\Omega^d$.[21,22]

[20] We simulate 10,000 periods and drop the first 1,000.

[21] We estimate the full variance-covariance matrix using bootstrap methods (Lee and Ingram, 1991; Bloom, 2009). This implies a total of 23×23 elements. Due to the high degree of nonlinearity of our model, we proceed to incorporate the off diagonal for efficiency reasons (Ruge-Murcia, 2012). See our Extended Online Appendix for alternative weighting matrices.

[22] We are particularly interested in the potential power of our model to match labor market dynamics. For that reason, we intentionally put relatively more weight on IRFs of ls to productivity z shocks and two labor market comovements, $\rho(w,y)$, and $\rho(ls,y)$. We also put more weight on getting the size of fluctuations, $var(y)$, right. More specifically, we divide the columns and rows associated with these specific moments of the full variance-covariance matrix by 10.

G Treating v_t as Unobservable vs. Observable: IRFs Comparison

In our model we have considered the presence of investment-specific technical change (ISTC) v_t, which alters the efficiency of investment and, hence, capital in efficiency units differs from real capital and follows

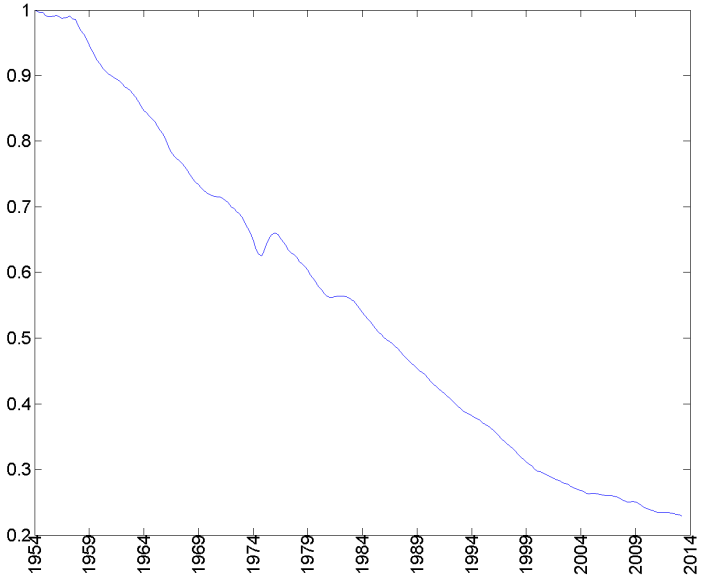
$$v_t i_t = k_{t+1} - (1 - \delta_0)k_t, \tag{66}$$

where δ_0 denotes a constant depreciation. In a one sector model v_t is identified with the inverse of the relative price of investment in terms of consumption. The construction of the relative price of investment, and the resulting investment in efficiency units and capital in efficiency units is described in detail in Appendix A. Figure G-1 shows the behavior of the variables created in Appendix A.

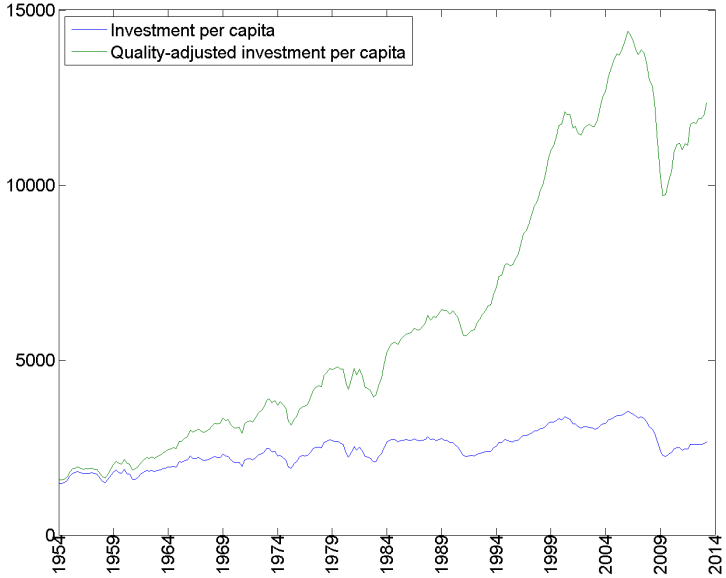
Here, we compare the model generated IRFs for the case where we treat the investment shock v_t as unobservable and structurally estimate the properties of ϕ_{v1}, ϕ_{v2} and υ_v versus the case where we treat the shocks to v_t as observable. In the latter case, we use an AR(2) process to estimate investment shocks and find $\phi_{v1} = 1.7428$, $\phi_{v2} =$-.7535 and $\upsilon_v =$.0029. As we see in Figure G-2 when we treat v_t shocks as observable we find similar IRFs to the case in which v_t shocks are treated as unobsersable, except for a somewhat weaker overshooting response of labor share to productivity shocks.

Figure G-1: Time-Series of the Relative Price of Investment and ISTC-Adjusted TFP, U.S. 1954.I-2012.III

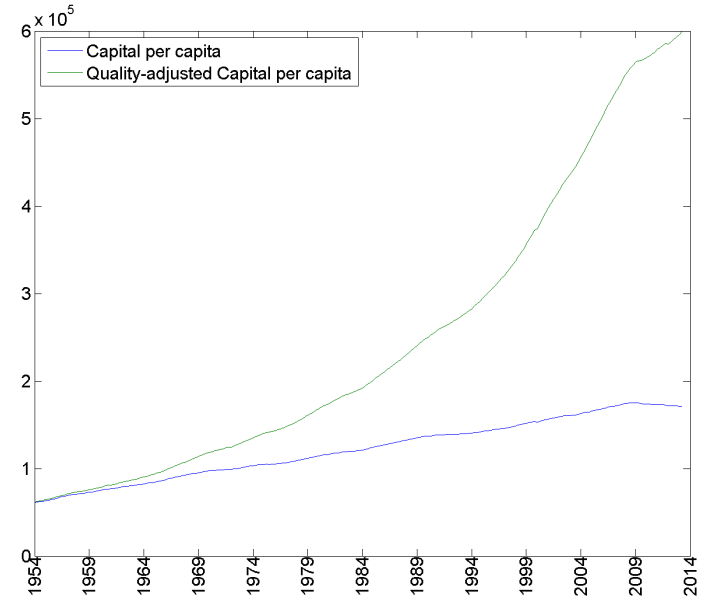
(a) Relative Price of Investment



(b) Quality-Adjusted Investment



(c) Quality-Adjusted Capital



(d) ISTC-Adjusted TFP

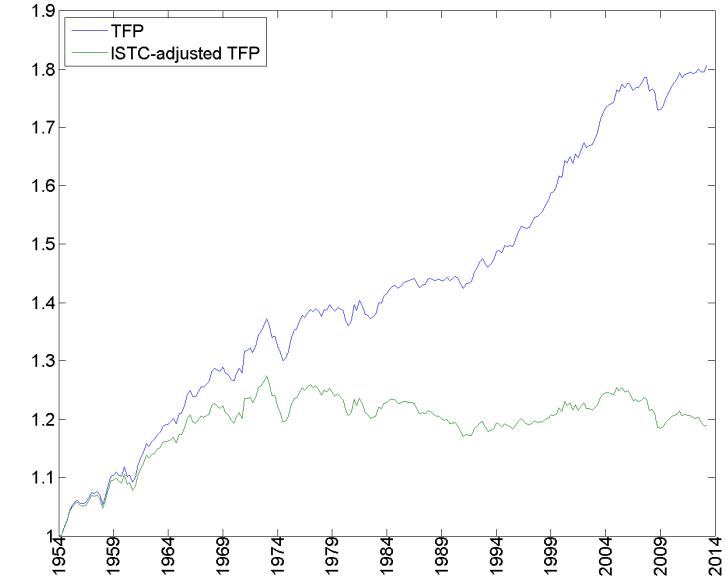
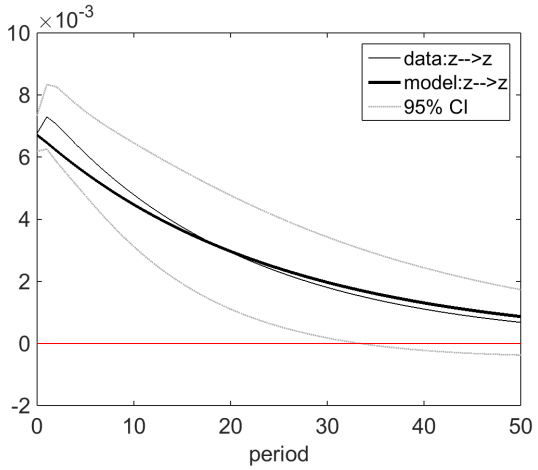
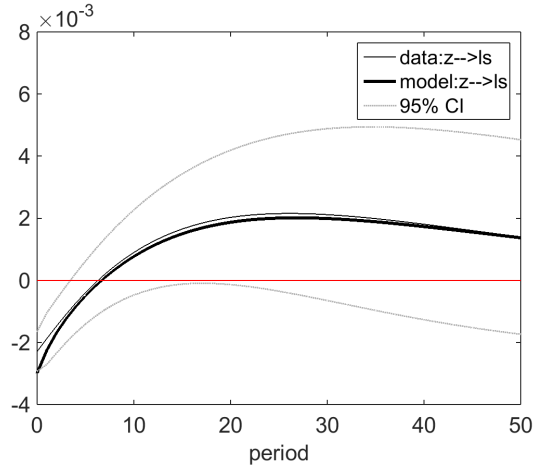
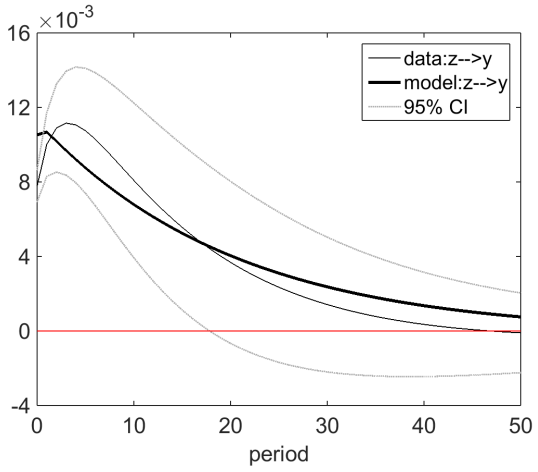


Figure G-2: IRFs of Labor Share and Output to z-shock: Unobservable v_t vs. Observable v_t

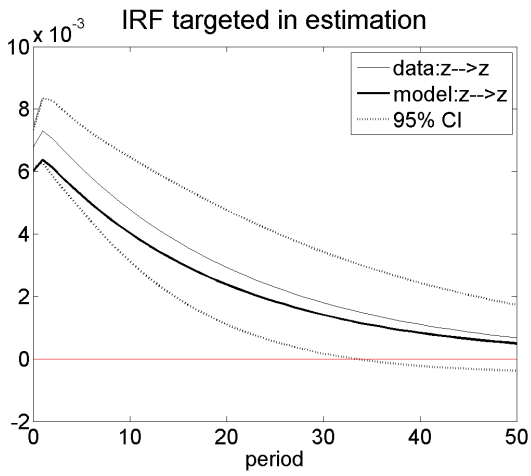
(a) Unobservable v_t

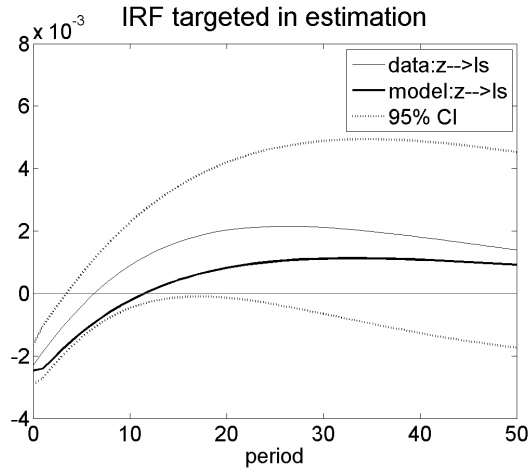


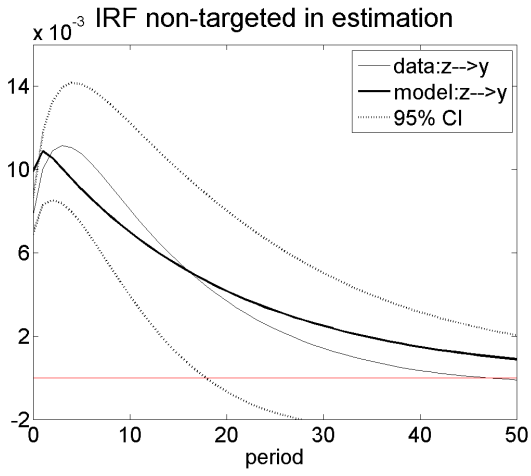




(b) Observable v_t







H Sources of Identification: Further Discussion

The identification of each and all of our structurally estimated parameters is discussed here. To do so we systematically study the sensitivity of our targeted moments (business cycle moments and IRF behavior) in response to deviations from our set of estimated parameters in a *ceteris paribus* fashion. This study helps us determine what targeted moment (or moments) is *locally* informative to identify a given estimated parameter.[23]

H.1 The Average of the Short-Run Elasticity of Substitution, σ

First, we study the sensitivity of targeted business cycle moments in response to deviations from our estimated (average) short-run elasticity of substitution, σ, in a *ceteris paribus* fashion. Factually, this is a discussion on the sources identifying σ in our model. We find that labor market behavior is very sensitive to average σ, which is extremely useful for its identification.

The top panel of Figure 4 shows the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to changes in average $\sigma \in$ (0,1.5].[24] The variance of output relative to the data, $\widetilde{v}_y$, starts at .18 with values of σ close to zero, after which it monotonically increases with σ reaching the variance of the data at σ =.60. The monotonicity of the response of $\widetilde{v}_y$ to σ implies that there is a unique σ that allows the model to match the cyclical variance of output in the data. The relative variance of labor share, $\widetilde{v}_{ls}$, displays a U-shaped behavior with respect to σ reaching its minimum close to $\widetilde{v}_{ls}$=.81 with σ=.79. Either decreasing or increasing σ increases $\widetilde{v}_{ls}$, reaching the data at σ =.55 and σ =1.06. However, by increasing σ beyond .79 we are driving $\widetilde{v}_y$ far from target. This trade off leaves the region with σ <.79 as the only possible candidate to match both output and labor share fluctuations at once, pushing our algorithm to unambiguously choose a short-run production function that is more complementary than Cobb-Douglas. Further, note that for σ <.60, $\widetilde{v}_{ls}$ increases rapidly away from the data, which our estimation algorithm will penalize. The bottom panel of Figure 4 shows the HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts. These correlations display a monotonic relationship with σ, making its identification transparent as there are respective unique values of σ that generate the correlations in the data. Increases in σ imply rapid increases in $\rho(w, y)$ and $\rho(ls, y)$ and rapid decreases in $\rho(lp, y)$. For instance, $\rho(ls, y)$ rises from -.96 to .72, covering almost its entire support (i.e., [-1,1]) between σ =.01 and σ =1.5 and hitting the data target around σ =.60. The correlations $\rho(w, y)$ and $\rho(lp, y)$ hit the data with σ around .60 and .63, respectively. The correlation $\rho(e, h)$ also displays an increasing monotonic relationship with σ after σ >.11, crossing its data counterpart around σ =.66. When we put all these pieces together, our estimation algorithm compromises at the estimated value of σ =.5981 which is depicted as an interior vertical line.

H.2 The Joint Dynamics of Labor-Augmenting Productivity Shocks, a_t, and the Elasticity of Substitution σ_t

Properties of the labor-augmenting productivity shocks a_t: ψ_{1a}, ψ_{2a} and υ_a. Both $\widetilde{v}_y$ and $\widetilde{v}_{ls}$ increase monotonically with ψ_{1a} hitting the data around ψ_{1a} =.95 (see top panels of Figure H-1). These relative variances increase rapidly after ψ_{1a} >1.00, that is, as the persistence of the a-shock, $|\psi_{1a} + \psi_{2a}|$, approaches a unit root

[23] The method of moments strategy for estimation allows, in addition to exploring the effects that an estimated parameter has on the loss function (i.e., a single statistic such as the score), for the thorough studying of how much a given parameter matters for any specific (set of) targeted moment(s), which is what we comprehensively explore in this Section. While we cannot theoretically ensure that the solution to our estimation algorithm is unique (i.e., a global minimum), we have found that our estimation tends to yield the same solution independently of our initial guess which we believe warrants the sensitivity analysis that we conduct here not only locally but also for relatively wide parameter ranges. Further, the fact that our discussion below shows that the response of our targeted moments to changes in the model estimated parameters is large and monotonic for most of our parameters suggests that our minimum is not in the neighborhood of any other minima.

[24] The relative variance $\widetilde{v_x}$ of a variable x is defined as the model-generated variance of that variable divided by its data counterpart; hence, a value of $\widetilde{v}_x$ =1.00 implies that the model exactly matches the data.

(where note that we are keeping ψ_{2a} =-.0595 constant). These variances decrease less strongly (in a convex fashion) when we reduce ψ_{1a}. The value of ψ_{2a} is somewhat less informative regarding $\widetilde{v}_y$ with a wide range of $\psi_{2a} \in$ (-.35,-.04) that brings the model to the data and $\widetilde{v}_{ls}$ with values around both -.75 and -.05 that match the data. Similarly, the labor market comovements are more informative about ψ_{1a} than ψ_{2a} (see bottom panels of Figure H-1). In particular, $\rho(w,y)$ and $\rho(ls,y)$ increase from relatively large negative values associated with small values of ψ_{1a} to the data counterparts around ψ_{1a} =.95. Out of the properties of a_t, the variance of its innovations, υ_a, is perhaps the most identifiable from relative variances and labor market comovements as all these moments are very sensitive to changes in υ_a. Increases in υ_a imply both (1) that $\widetilde{v}_y$ and $\widetilde{v}_{ls}$ monotonically increase from values around .3 at υ_a =.00 to data values around υ_a =.009 and (2) that $\rho(w,y)$, $\rho(lp,y)$ and $\rho(ls,y)$ also increase monotonically from values close to -1.00 at υ_a =-.00 to data values around υ_a =.009.

The top panel in Figure 5 shows the effects of ψ_{1a}, ψ_{2a} and υ_a on the model-generated IRF of labor share in response to productivity, and the bottom panel of Figure 5 shows the contour sets associated with this IRF. While values below ψ_{1a} <.90 generate a strong peak response of labor share, they are also associated with initial responses outside the 95% CI in the data. Further, values above ψ_{1a} >.96 rapidly drop the peak response of labor share, which bounds our estimated ψ_{1a} between .90 and .96. The IRF of labor share to productivity is highly sensitive to ψ_{2a}. With ψ_{2a} =-.80 the overshooting reaches a peak of 0.0047 log points (almost twice its data counterpart) but it rises too fast (the peak occurs before the 10th period after a productivity impact, while it does so about 26 periods after impact in the data) and dies out too quickly as well due to the implied lower persistence of a $|\psi_{1a}+\psi_{2a}|$ = |.9536-.80| ≈.20. Finally, values of υ_a below .0080 imply an initial response that is relatively too high, -.0015, with respect to the data, -.0021, suggesting a reasonable parameterization for υ_a must be above .80. Our algorithm compromises with ψ_{1a} =.9536, ψ_{2a} =-.0595, and υ_a =.0090.

Properties of the elasticity of substitution shocks: $\psi_{1\sigma}$, $\psi_{2\sigma}$ and υ_σ. The effects of these parameters on the set of targeted unconditional moments are plotted in Figure H-2 and on the targeted IRF of labor share to productivity innovations in Figure H-3. For values below $\psi_{1\sigma}$ <1.00 and $\psi_{2\sigma}$ <-.20, $\widetilde{v}_y$ is fairly constant and close to the data target, while it increases rapidly and far away from data above those values, a feature that our estimation algorithm will penalize. The behavior of $\widetilde{v}_{ls}$ is more informative as it monotonically increases from values around .6 associated with low values of $\psi_{1\sigma}$ or $\psi_{2\sigma}$ to uniquely match the data at around $\psi_{1\sigma}$ =1.00 and $\psi_{2\sigma}$ =-.20. As opposed to the properties of the a_t shock, it is now the behavior of $\rho(ls,y)$ and $\rho(lp,y)$ (instead of $\rho(w,y)$) that helps determine a unique value for $\psi_{1\sigma}$ around .98. The identification of $\psi_{2\sigma}$ is much less obvious from labor market comovements. The behavior of the IRFs of labor share and associated contour maps in Figure H-3 suggest that values around $\psi_{1\sigma}$ =.94 generate the earliest possible overshooting of .0020 log points at the 30th period, while values below -.4 for $\psi_{2\sigma}$ generate earlier overshooting and a higher peak between the 25th and 27th quarter, which conforms to the data. Finally, the set of unconditional moments is not very informative about the identification of υ_σ as all targeted moments remain rather flat to υ_σ changes, and the IRF of labor share to productivity innovations is also fairly constant across values of υ_σ. Our algorithm compromises to $\psi_{1\sigma}$ =.9830, $\psi_{2\sigma}$ =-.2049, and υ_σ =.0145.

The effects of labor-augmenting productivity shocks a_t on σ_t: $\psi_{1a\sigma}, \psi_{2a\sigma}$ and $\upsilon_{a\sigma}$. These parameters describe the effect of labor-augmenting productivity shocks on σ_t. The importance of these parameters to the set of targeted business cycle moments is shown in Figure H-4 and to the IRF of labor share in Figure H-5. The behavior of unconditional moments with respect to $\psi_{1a\sigma}$ is highly informative about $\psi_{1a\sigma}$. Both $\widetilde{v}_y$ and $\widetilde{v}_{ls}$ pin down a unique and similar value for $\psi_{1a\sigma}$ that matches the data around -7.0. In this case, it is $\widetilde{v}_{ls}$ that makes $\psi_{1a\sigma}$ more identifiable with a large swing that goes from more than 2.00 to .46 between $\psi_{1a\sigma}$ =-10.0 and $\psi_{1a\sigma}$ =-2.86. This is consistent with the information from the correlations of labor market variables with output, in particular, that of labor share and output that covers almost half of its support uniquely crossing its data counterpart around $\psi_{1a\sigma}$ =-7.0 and that of labor productivity and output that also presents a large decline from .39 in $\psi_{1a\sigma}$ =-3.06 to -.14 in $\psi_{1a\sigma}$ =-10.0. Similarly, high identification properties are displayed by $\psi_{2a\sigma}$ with $\widetilde{v}_{ls}$, $\rho(lp,y)$ and $\rho(ls,y)$ being the most informative moments. The contour sets of the IRFs reveal that values $\psi_{1a\sigma}$ <-5.0 and values of $\psi_{2a\sigma}$ that are outside the interval (2.0,4.5) substantially delay the period at which labor share overshoots its own mean after a productivity innovation, moving equilibrium allocations away from target; for example, with $\psi_{1a\sigma}$ =-1.00 the overshooting occurs at the 15th quarter after impact and with $\psi_{2a\sigma} = -5$ at the 14th quarter while this is occurring at around 6th period in the data. That is, $\psi_{1a\sigma}$ and $\psi_{2a\sigma}$ have strong

implications for determining the period in which labor share overshoots. We will come back to the importance of $\psi_{1a\sigma}$ and $\psi_{2a\sigma}$ in generating the IRF of labor share in Section 4. Finally, the case of $\upsilon_{a\sigma}$ is different as neither the business cycle moments nor the IRF of labor share seems informative about the identification of $\upsilon_{a\sigma}$. Our algorithm compromises to $\psi_{1a\sigma}$ =-6.8144, $\psi_{2a\sigma}$ =2.9812, and $\upsilon_{a\sigma}$ =-.4827.

H.3 Properties of the Investment Shocks, v_t

Regarding with unconditional moments, see Figure H-6. It is $\widetilde{\upsilon}_{ls}$ and $\rho(ls,y)$ that pin down a value of ψ_{1v} slightly above 1.6 after which (1) labor share rapidly increases from .80 at ψ_{1v} =1.65 to values above 1.00 at ψ_{1v} = 1.68 and (2) $\rho(ls,y)$ rises from -.60 to -.34 for those respective values of ψ_{1v}. Likewise,ψ_{2v} is evaluated in the range of [-0.8,-0.675], holding ψ_{1v} fixed at 1.6763. We find that $\widetilde{\upsilon}_{y}$ and $\widetilde{\upsilon}_{ls}$ decrease as ψ_{2v} increases from low values to -0.68. The labor market comovements are also very sensitive to ψ_{2v}. It is evident from the figure that $\rho(w,y)$, $\rho(lp,y)$, and $\rho(ls,y)$ monotonically increase in a convex manner for ψ_{2v} <-0.685, whereas $\rho(e,h)$ monotonically decreases up to the same point. Putting these two pieces together for ψ_{1v} and ψ_{2v} justifies the high estimated persistence of v_t. Further, υ_v is highly identifiable with $\widetilde{\upsilon}_{y}$ and $\widetilde{\upsilon}_{ls}$ monotonically increasing as υ_v increases, matching their data counterparts around υ_v =.0035. The effects of υ_v on the labor market comovements are also very strong with $\rho(w,y)$ and $\rho(lp,y)$ monotonically declining as υ_v increases, and with $\rho(ls,y)$ and $\rho(e,h)$ increasing with υ_v.

Regarding with dynamics of labor share, our findings are that the properties of v_t strongly affect the IRF of labor share (see these IRFs in the top panels of Figure H-7 and in the associated contour maps in the bottom panel of that Figure).[25] It is clear that while ψ_{1v}, ψ_{2v} and υ_v do not affect the initial response of labor share or its overshooting (which we have seen is guided by $\psi_{1a\sigma}$ and $\psi_{2a\sigma}$), they do have an effect on how much an existing overshooting lasts. Lowering values of ψ_{1v}, ψ_{2v} and υ_v in all cases reduces the size and length of the overshooting response of labor share. Our algorithm compromises to ψ_{1v} =1.6763, ψ_{2v} =-.6808, and υ_v =.0035.[26] The power that v_t has on this response of labor share is tightly linked to the presence of fluctuations in σ_t.

H.4 Properties of the Home-Productivity Shocks, b_t

We find that the home-productivity shocks, b_t, is crucial in generating the cyclical correlation between employment per capita and hours per worker, $\rho(e,h)$, but otherwise, the rest of the targeted moments (including IRFs) are unresponsive to b_t. See Figures H-8 and H-9.The model implied behavior of $\rho(e,h)$ suggests a value of ψ_{1b} slightly above one and ψ_{2b} slightly below zero, implying a relatively high persistence for b_t. It is υ_b that has the strongest implications for $\rho(e,h)$, ranging from $\rho(e,h)$ =1.00 at υ_b =.0000 to $\rho(e,h)$ =-.39 at υ_b =.0100 and hitting the data target around υ_b =.0030. Our algorithm compromises to ψ_{1b} =1.0370, ψ_{2b} =-.0619, and υ_b =.0031.[27]

H.5 The Elasticity of Capital Depreciation with Respect to Utilization, ζ

While the elasticity of capital depreciation with respect to utilization, ζ, is not identified from the long-run (steady-state) conditions of the model, our targeted moments show a strong sensitivity to ζ.[28] This makes this elasticity highly identifiable from the full structure of our model. We find that as ζ increases, the variance of output increases monotonically, and the variance of the labor share decreases monotonically (see Figure H-10 in our online appendix). While the variance of labor share is matched at values of ζ close to zero, the variance of

[25] Note the high estimated persistence of investment shocks, $|\psi_{1v}+\psi_{2v}|$ =.9955, implies that our sensitivity analysis in Figure H-7 is restricted, in order to preserve stationarity, to values below the structurally estimated parameters ψ_{1v} and ψ_{2v}.

[26] See our Appendix H.3 for further details on the identification of the properties of v_t.

[27] See Figure H-8 and Figure H-9 in the online appendix.

[28] The range of ζ chosen in previous studies is large: Basu and Kimball (1997) estimates $\zeta = 1$, while Christiano et al. (2005) set this value to be 100. Ríos-Rull et al. (2012) explore several values {0,0.2,1,2,5}, just as King and Rebelo (1999) did for values between 0 and 10.

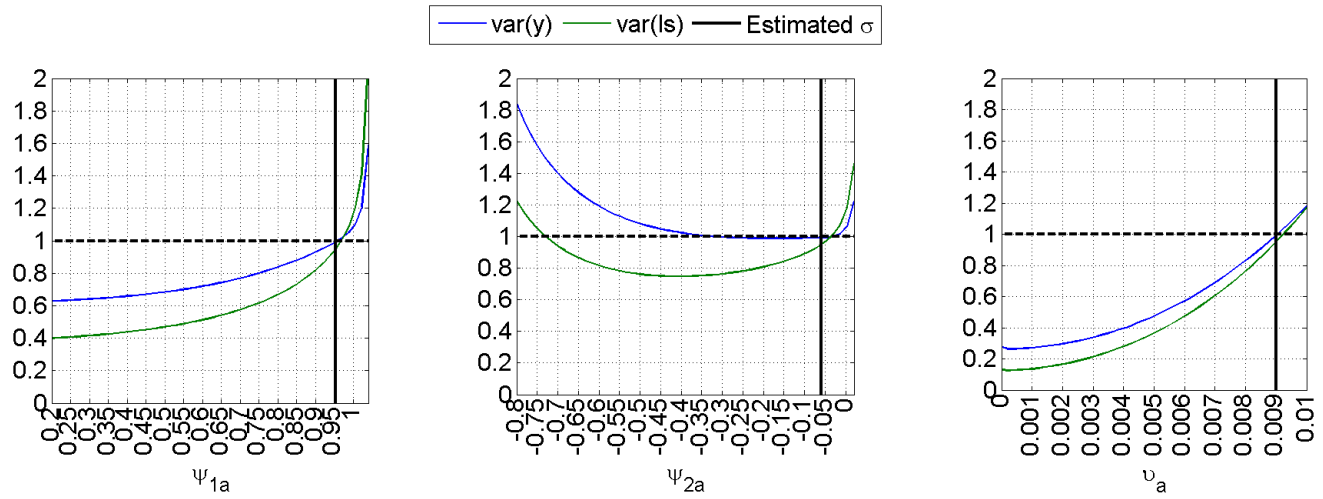
output matches the data at ζ =.1270. Further, the correlations of labor market variables with output all display monotonic relationships in ζ. Hence, there are unique values of ζ that deliver each of these comovements: ζ =.18 for $\rho(w, y)$, ζ =.02 for $\rho(lp, y)$, ζ =.10 for $\rho(ls, y)$ and ζ =.21 for $\rho(e, h)$. Taking into account that we put a higher weight on getting the size of fluctuations of output, $\rho(w, y)$ and $\rho(ls, y)$ right, our algorithm facing the trade-offs we just described compromises with a ζ =.1191. Lastly, note that once ζ is chosen, then we can use the steady-state condition,

$$r_0 = \delta_1 \left(1 + \frac{1}{\zeta}\right), \tag{67}$$

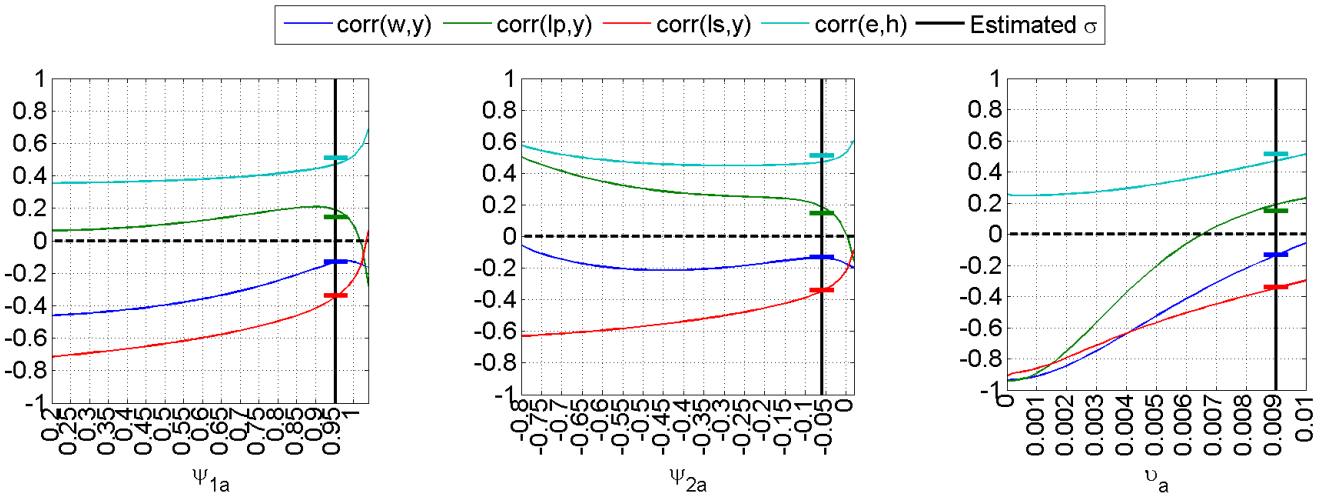
to find the value of δ_1= .0034 associated with the long-run interest rate r_0=.0324.

Figure H-1: Identification of ψ_{1a}, ψ_{2a} and υ_a: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



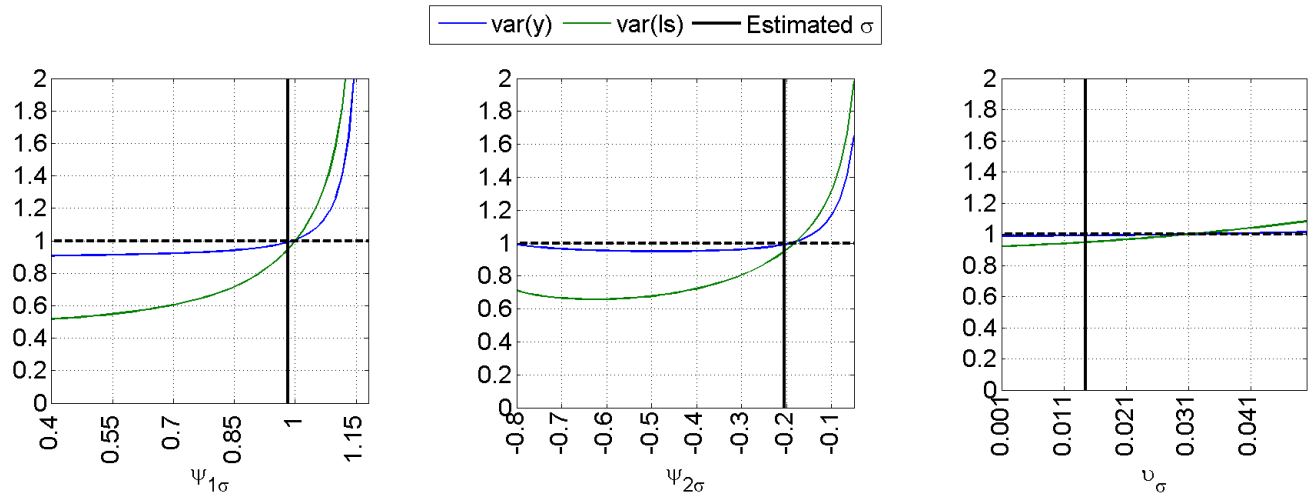
(b) Correlations with Output



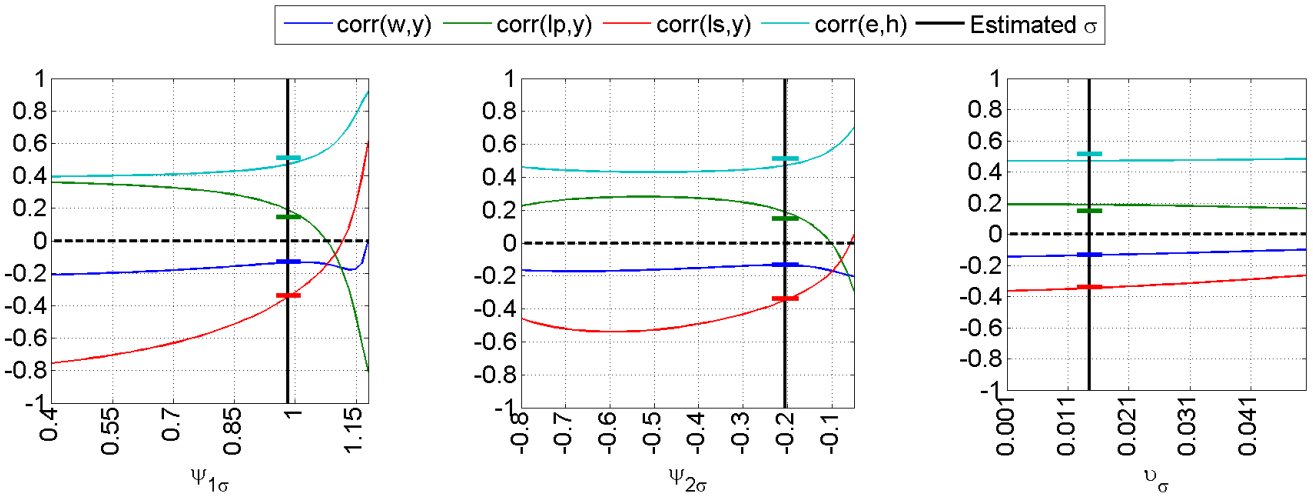
Notes: The top panels show the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the properties of the labor-augmenting technical shocks: ψ_{a1} in top-left panel (a), ψ_{a2} in top-center panel (b) and υ_a in top-right -panel (c). The bottom panels show the associated HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

Figure H-2: Identification of $\psi_{1\sigma}$, $\psi_{2\sigma}$ and υ_{σ}: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



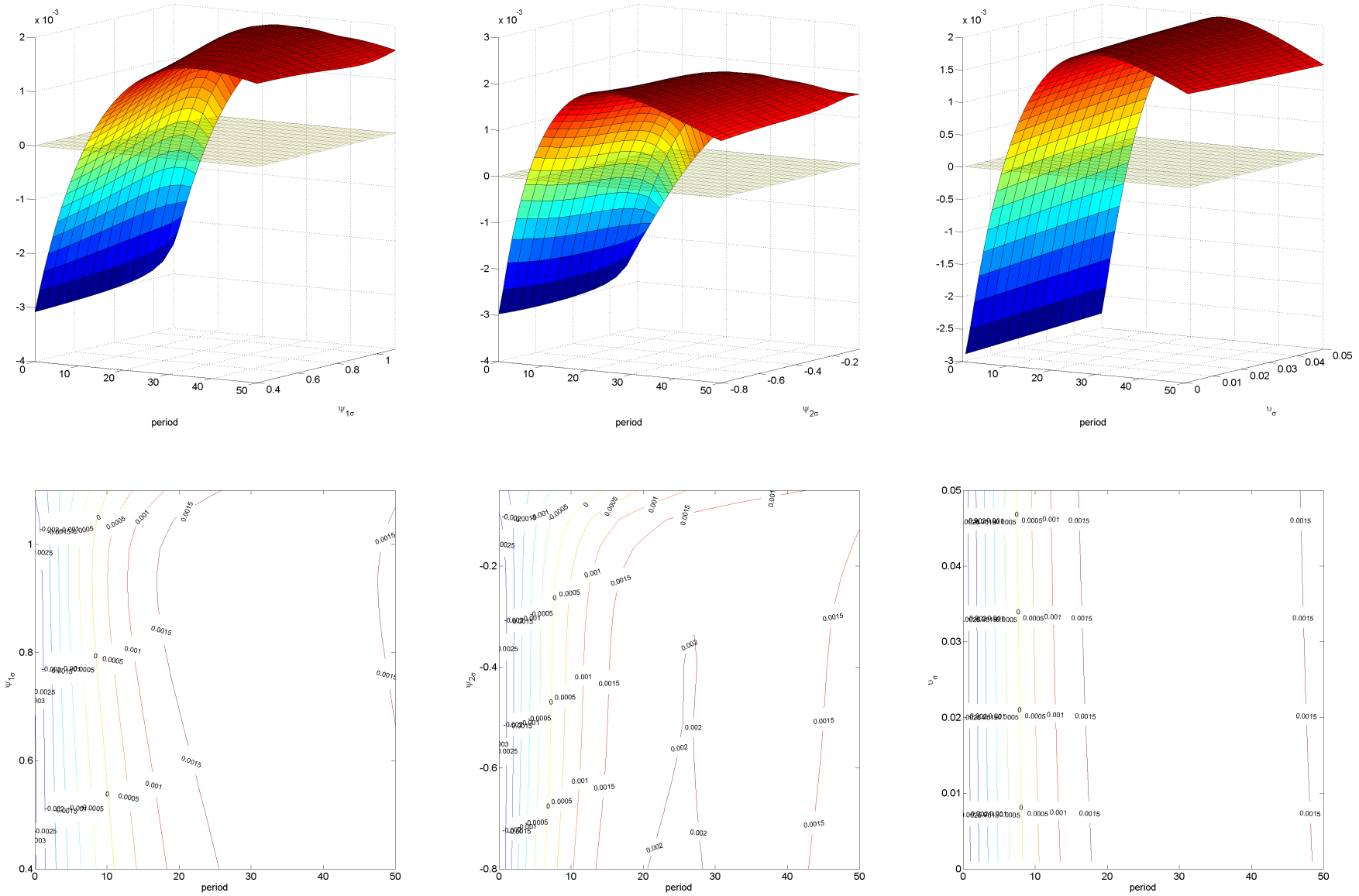
(b) Correlations with Output



Notes: The top panels show the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the properties of the shocks to the elasticity of substitution: $\psi_{\sigma 1}$ in top-left panel (a), $\psi_{\sigma 2}$ in top-center panel (b) and υ_{σ} in top-right -panel (c). The bottom panels show the associated HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

Figure H-3: Identification of $\psi_{\sigma 1}$, $\psi_{\sigma 2}$ and υ_{σ}: IRFs of Labor Share to Productivity shocks

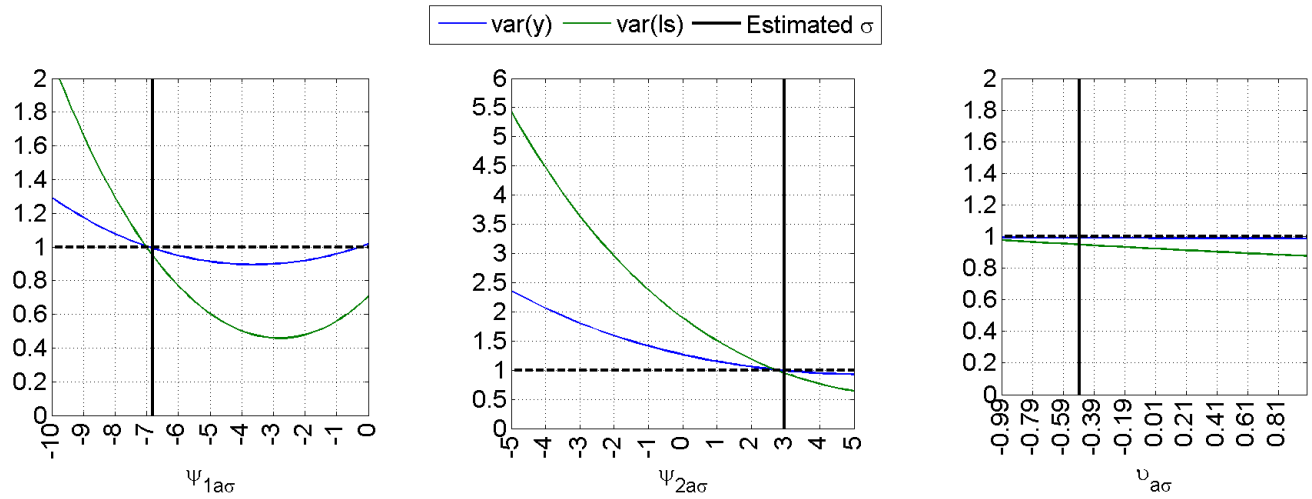
(a) $\psi_{\sigma 1}:\ z \to ls$ (b) $\psi_{\sigma 2}:\ z \to ls$ (c) $\upsilon_{\sigma}:\ z \to ls$



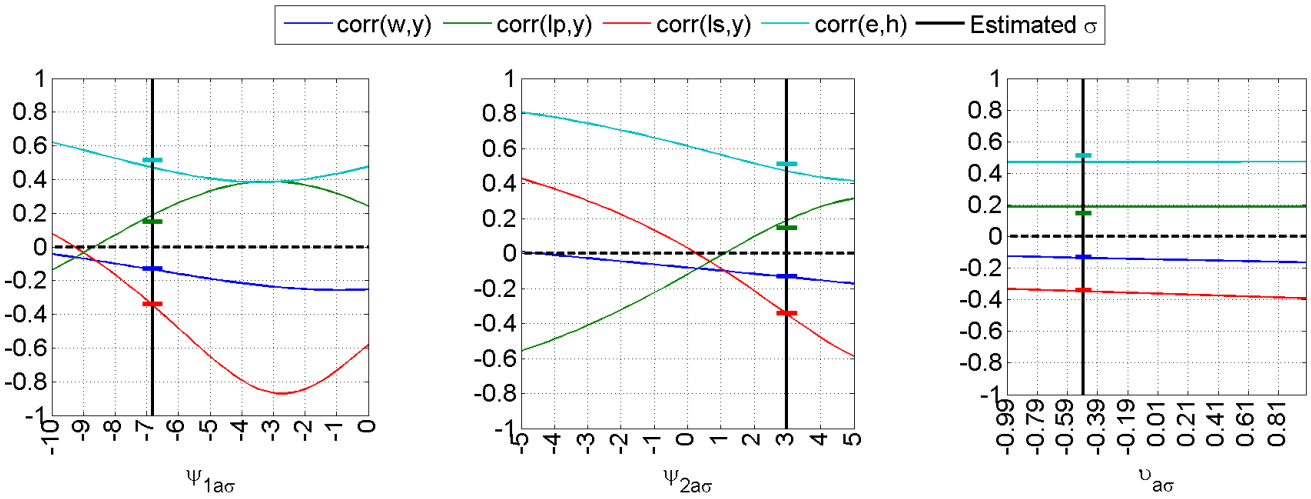
Notes: The top panels show the model IRF of labor share in response to productivity shocks for *ceteris paribus* changes in the properties of the shocks to the elasticity of substitution: $\psi_{\sigma 1}$ in top-left panel (a), $\psi_{\sigma 2}$ in top-center panel (b) and υ_{σ} in top-right -panel (c). The bottom panels show the associated contour sets of the IRFs of interest.

Figure H-4: Identification of $\psi_{a\sigma 1}$, $\psi_{a\sigma 2}$ and $\upsilon_{a\sigma}$: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



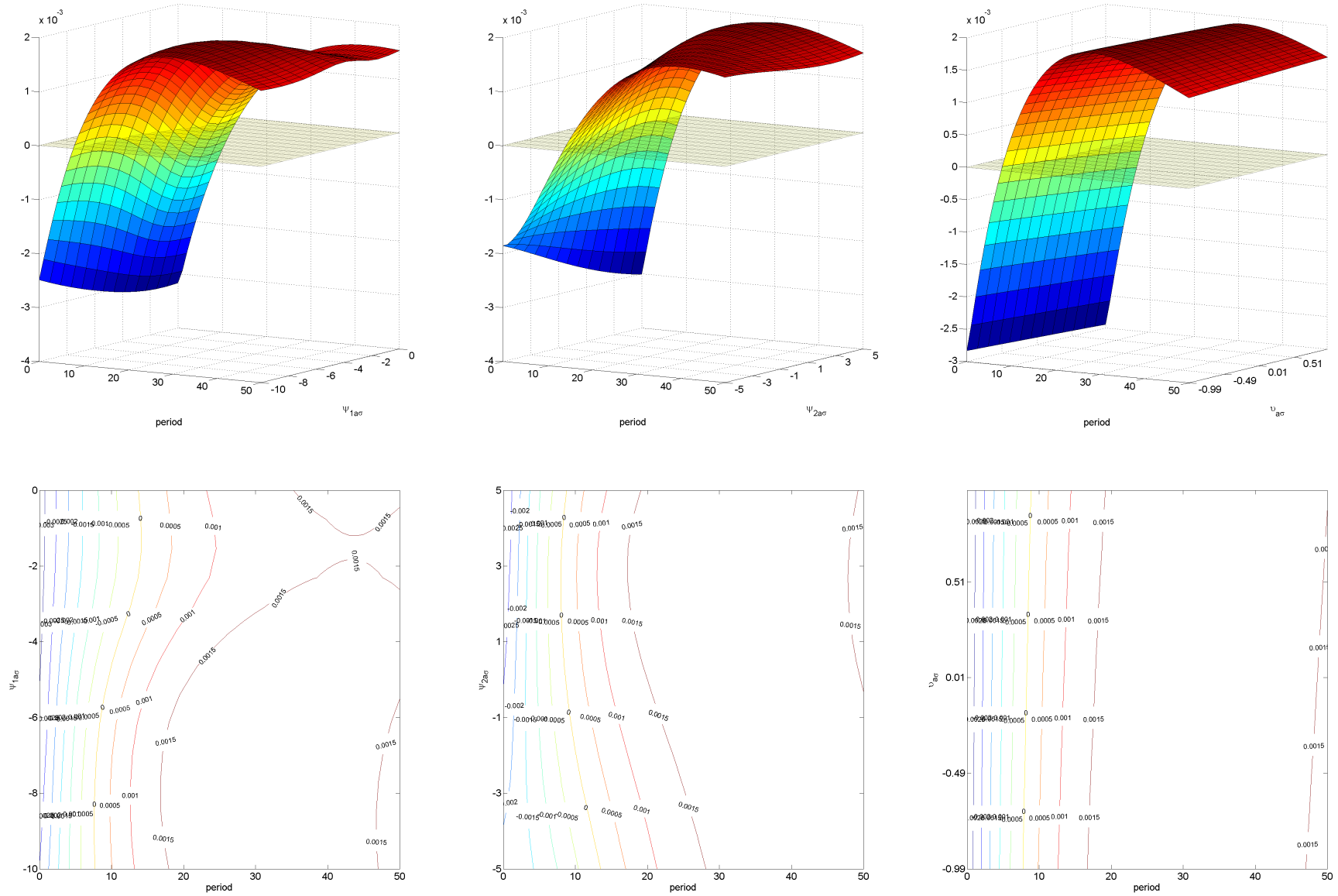
(b) Correlations with Output



Notes: The top panels show the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the properties of the dynamic effects of labor-augmenting technical shocks on the elasticity of substitution: $\psi_{a\sigma 1}$ in top-left panel (a), $\psi_{a\sigma 2}$ in top-center panel (b) and $\upsilon_{a\sigma}$ in top-right -panel (c). The bottom panels show the associated HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

Figure H-5: Identification of $\psi_{1a\sigma}$, $\psi_{2a\sigma}$ and $\upsilon_{a\sigma}$: IRFs of Labor Share to Productivity shocks

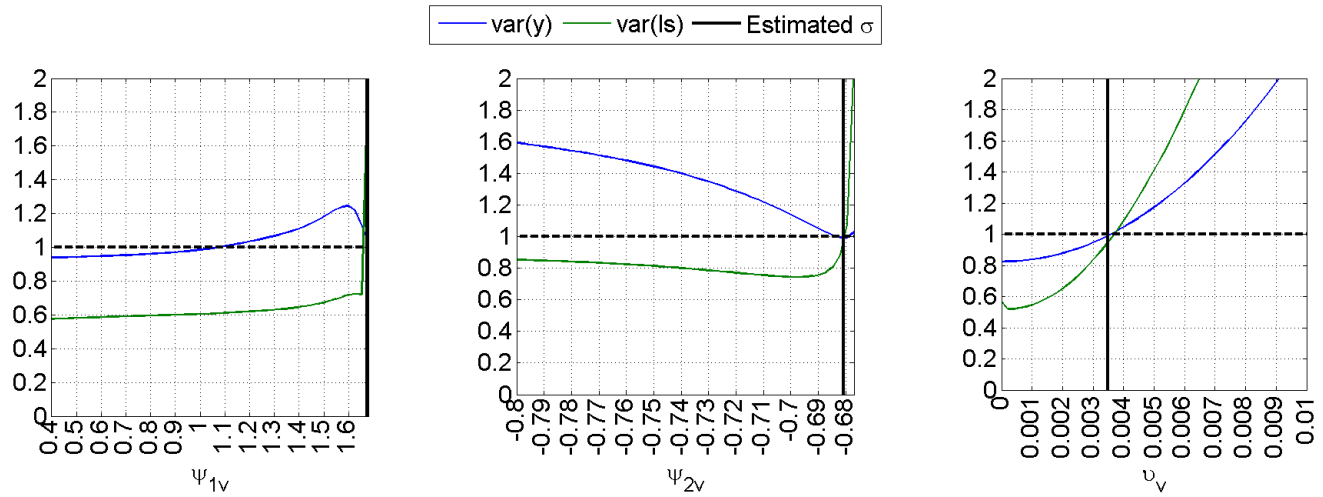
(a) $\psi_{a\sigma 1}:\ z \to ls$ (b) $\psi_{a\sigma 2}:\ z \to ls$ (c) $\upsilon_{a\sigma}:\ z \to ls$



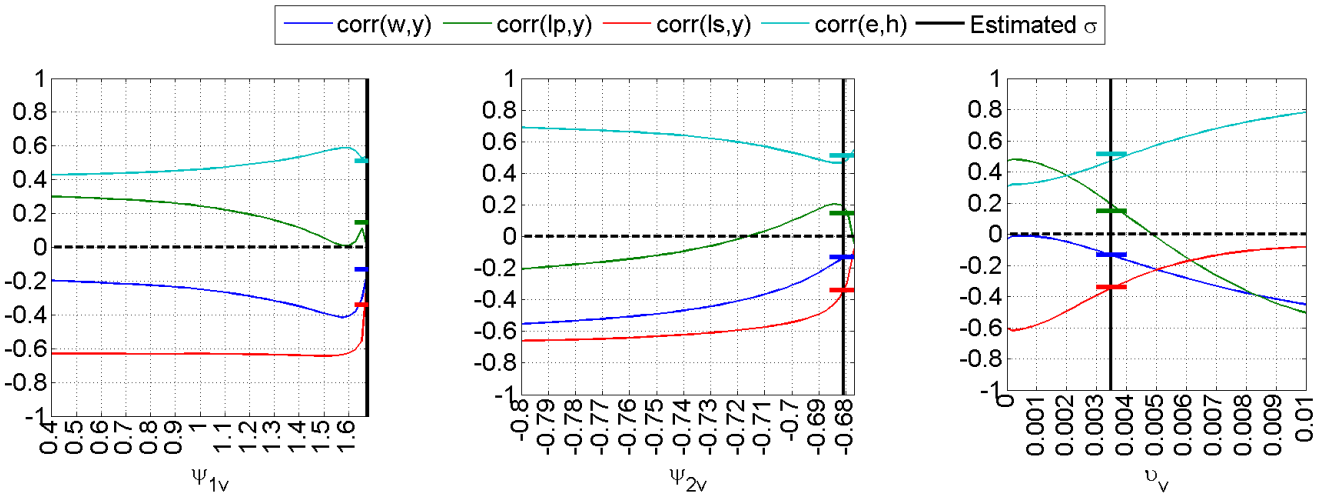
Notes: The top panels show the model IRF of labor share in response to productivity shocks for *ceteris paribus* changes in the properties of the dynamic effects of labor-augmenting technical shocks on the elasticity of substitution: $\psi_{a\sigma 1}$ in top-left panel (a), $\psi_{a\sigma 2}$ in top-center panel (b) and $\upsilon_{a\sigma}$ in top-right -panel (c). The bottom panels show the associated contour sets of the IRFs of interest.

Figure H-6: Identification of ψ_{1v}, ψ_{2v} and υ_v: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



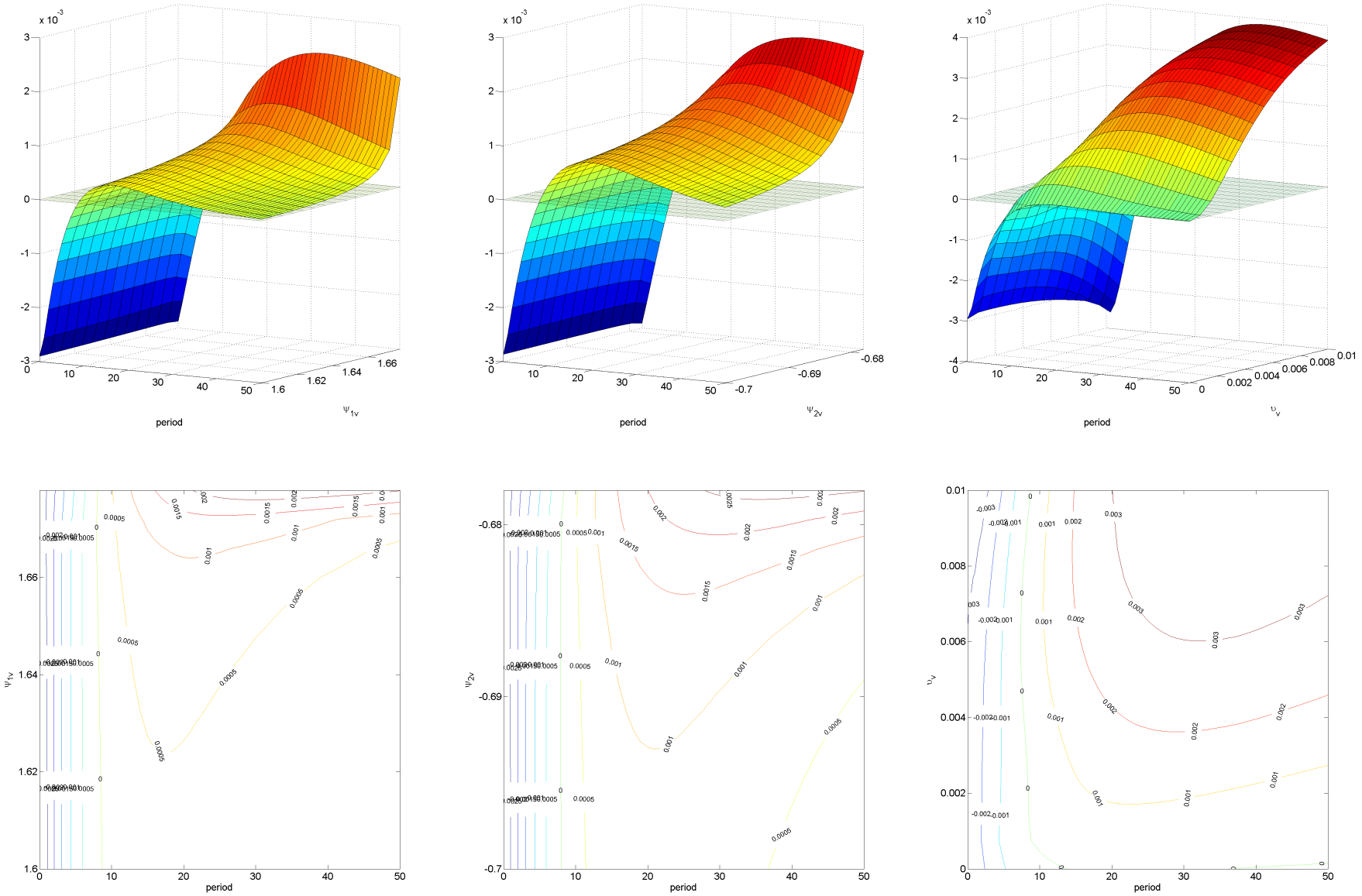
(b) Correlations with Output



Notes: The top panels show the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the properties of investment shocks: ψ_{v1} in top-left panel (a), ψ_{v2} in top-center panel (b) and υ_v in top-right -panel (c). The bottom panels show the associated HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

Figure H-7: Identification of ψ_{1v}, ψ_{2v} and υ_v: IRFs of Labor Share to Productivity shocks

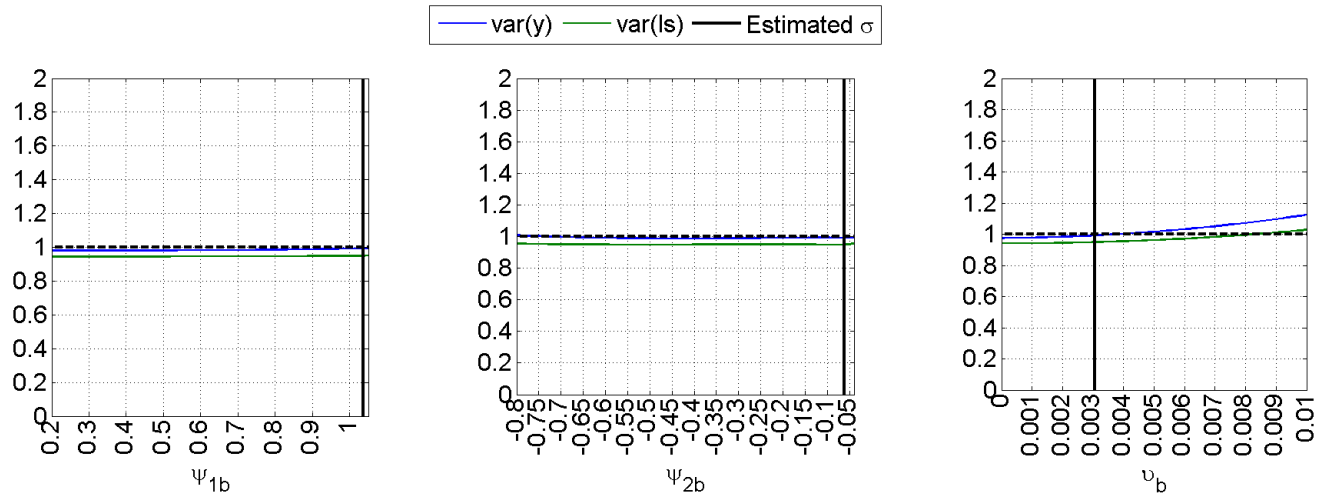
(a) ψ_{1v} : $z \to ls$ (b) ψ_{2v} : $z \to ls$ (c) υ_v : $z \to ls$



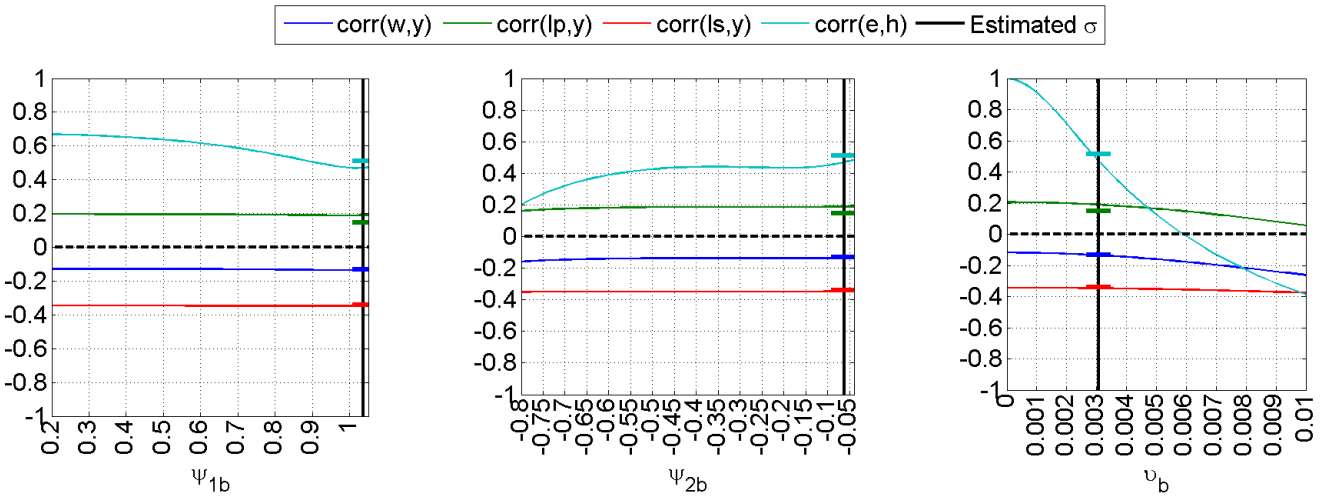
Notes: The top panels show the model IRF of labor share in response to productivity shocks for *ceteris paribus* changes in the properties of the dynamic effects of ISTC: ψ_{v1} in top-left panel (a), ψ_{v2} in top-center panel (b) and υ_v in top-right -panel (c). The bottom panels show the associated contour sets of the IRFs of interest.

Figure H-8: Identification of ψ_{1b}, ψ_{2b} and υ_b: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



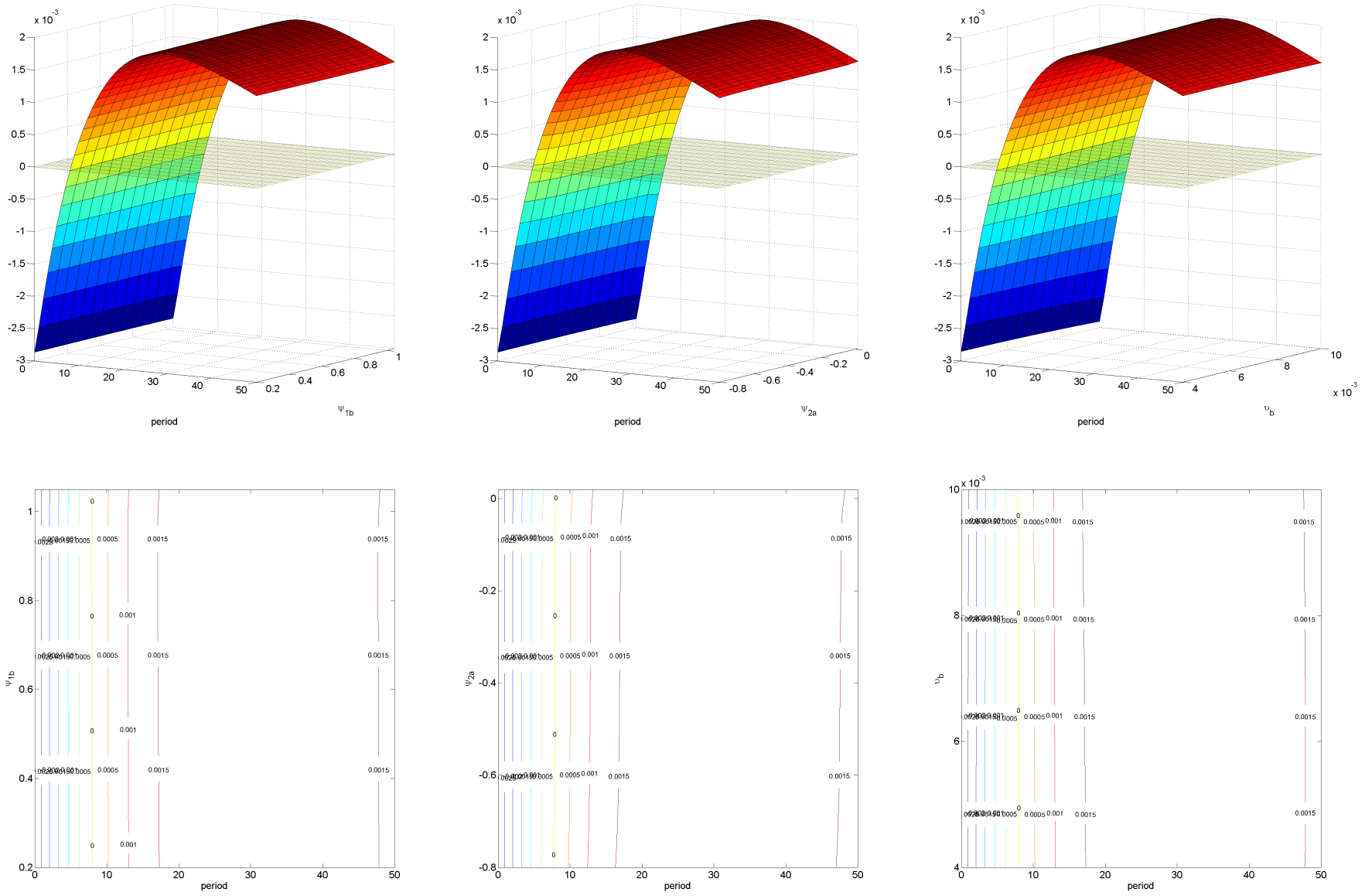
(b) Correlations with Output



Notes: The top panels show the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the properties of home-productivity shocks: ψ_{b1} in top-left panel (a), ψ_{b2} in top-center panel (b) and υ_b in top-right -panel (c). The bottom panels show the associated HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

Figure H-9: Identification of ψ_{1b}, ψ_{2b} and υ_b: IRFs of Labor Share to Productivity shocks

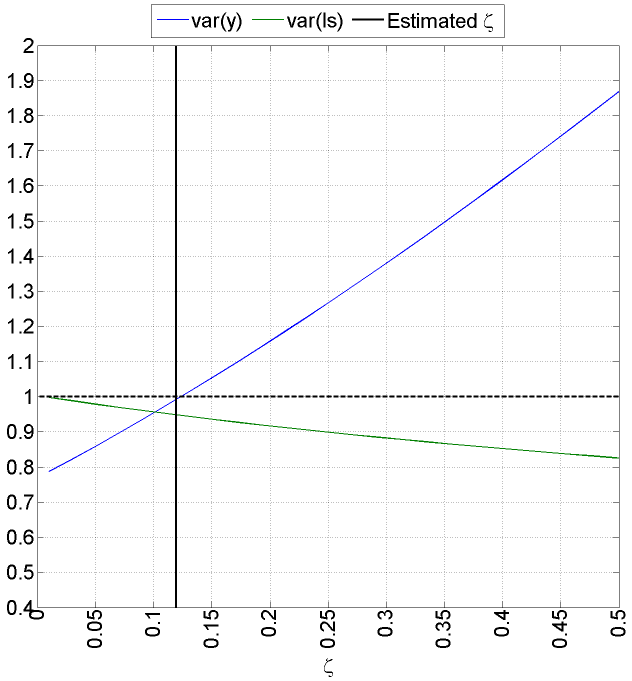
(a) ψ_{b1} : $z \to ls$ (b) ψ_{b2} : $z \to ls$ (c) υ_b : $z \to ls$



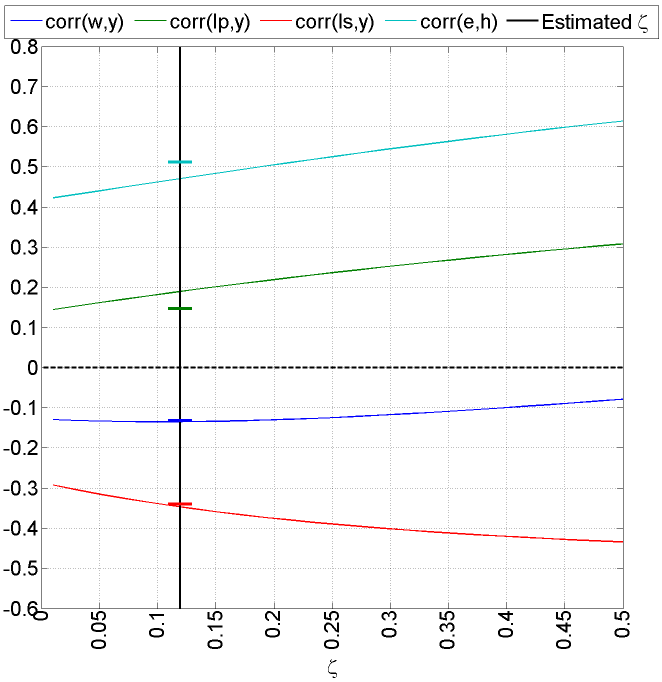
Notes: The top panels show the model IRF of labor share in response to productivity shocks for *ceteris paribus* changes in the properties of home-productivity shocks: ψ_{b1} in top-left panel (a), ψ_{b2} in top-center panel (b) and υ_b in top-right -panel (c). The bottom panels show the associated contour sets of the IRFs of interest.

Figure H-10: Identification of ζ: Sensitivity of Targeted Moments

(a) Variances of Model Variables Divided by Data Counterparts



(b) Correlations with Output



Notes: Panel (a) shows the behavior of model HP-filtered variances of output and labor share (relative to data counterparts) in response to *ceteris paribus* changes in the elasticity of capital deprectiation with respect to utilizatioin, ζ. Panel (b) shows the HP-filtered correlations of model labor market variables with output—tick marks indicate data counterparts.

I Getting Consumption Right

In Section 5, we pointed out the negative correlation between output and consumption that business cycle models with investment shocks or government shocks typically deliver. Here, we explore four alternatives to get the behavior of consumption right. We also discuss under which of these alternatives our labor market results are robust to the relationship between consumption and output.

I.1 Shutting Down Investment Shocks and Government Shocks

Our discussion in Section 6.2 highlighted the fact that the introduction of both investment shocks and government shocks drives the correlation of consumption and output to negative values. Here we take our benchmark NCES-CD model and shut down investment shocks and government shocks. The results of this exercise are in the first set of three columns in Table I-1. We find that while removing these shocks helps generate a positive correlation between consumption and output of .50, this comes at the cost of small fluctuations in consumption and a worsening in the labor market behavior. Regarding the labor market, we observe that the correlation of wages and output rises from our benchmark of -.14 to .57, away from target, and that the correlation of labor productivity with output is .80, while it is .19 in our benchmark NCES model and .15 in the data. Further, while labor share is countercyclical with $\rho(ls, y)$ =-.63, this figure is (in absolute terms) two times larger than our benchmark results (or the data). These results align with our discussion in Section 6.2 in which we observe that the introduction of investment shocks to a model with only productivity (and σ_t) shocks helps decrease the correlation of wages and labor productivity with output and that the introduction of government shocks implies similar insights with further reductions in the correlation of wages and labor productivity with output. That is, these effects revert, once we remove investment and government shocks. Overall, this implies that simply removing the effects of investment and government shocks is not a useful solution to the consumption puzzle, as it comes at the cost of labor market dynamics.

I.2 Adding Adjustment Costs

The second set of three columns in Table I-1 presents the business cycle moments when we introduce investment adjustment costs with a standard convex function into our NCES-CD benchmark model.[29,30] Our main finding is that adjustment costs are successful at reverting the negative correlation between consumption and output without distorting labor market results. Here, we find that consumption co-moves positively with output with a cyclical correlation of .68 (being .93 in the data, see Table 4). Further the volatility of consumption also increases, accounting for 74% of the fluctuations observed in the data. The effects of adjustment costs on our benchmark labor market results are minor with wages and labor productivity still presenting low correlations with output, and labor share being countercyclical with correlation with output of -.22, close to the data target.

[29] Recent DSGE models that incorporate investment adjustment costs estimate the adjustment cost parameter without using a specific parametric form (e.g., Christiano et al. (2005)). In those settings, adjustment costs are parameterized in a linearized model. But as we discussed in Appendix D, we have to rely on a nonlinear solution method in our framework, and hence, we need to specify the functional form of the adjustment costs. We choose the most standard formulation of convex costs $\frac{\phi}{2}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2$, which follows Hayashi (1982) (see Sargent (1978) for an earlier version). Note that this functional form indeed satisfies the estimation criteria in Christiano et al. (2005). Given that we are not estimating the value of the adjustment cost parameter, we choose a value of ϕ from the estimated second derivative in Christiano et al. (2005), which implies ϕ =26.04. This value is close to the value chosen in Hayashi (1982), ϕ =20.

[30] Given that we introduce a new feature in the model, we re-estimate the model. For this same reason, we also re-estimate the extended models in the next subsections where we incorporate either habit persistence or preferences without wealth effects.

I.3 Using Preferences with Habit Persistence

Habits in consumption alter the intertemporal substitution of consumption, which can help correct for the negative correlation imposed by investment and government shocks (see discussions in Jaimovich and Rebelo (2006) and Christiano et al. (2010)). Here, we introduce in our NCES-CD benchmark the preferences proposed in Boldrin et al. (2001) with persistence parameter .90. Our findings are in the third set of three columns in Table I-1. We find that while habit persistence improves the correlation of consumption, which increases to -.09, the overall size of aggregate fluctuations decreases in the model; in particular, the cyclical variance of consumption drops to .06. The effects of habit persistence on labor market behavior suggest that it does not distort the variances and correlations of labor market variables.

I.4 Using Preferences without Wealth Effects: Greenwood et al. (1988)

The fact that government shocks reduce the correlation of output and consumption is largely due to the presence of wealth effects. Here we explore the use of preferences without wealth effects introduced in Greenwood et al. (1988). While these preferences are not consistent with BGP (see King et al. (2002)), there is no theoretical reason undermining their use in the short run. This is an analogous argument to the one that we have proposed to use our short-run component of production that differs from its long-run counterpart associated with BGP (see Section 2.1). For our exercise we use the following specification of GHH preferences,

$$u(c,h) = \frac{1}{1-\gamma}\left(c_t - \kappa\frac{(d_t e h_t)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}}\right)^{1-\gamma}, \tag{68}$$

which we impose on our NCES-CD model. Note that eh_t is employment per capita as here we do not distinguish between intensive and extensive margins of labor supply. We calibrate the disutility parameter, κ, by taking ν =0.72 (as in our benchmark calibration) and γ =2 as in Greenwood et al. (1988). Our main results are in the fourth set of three columns in Table I-1. We find that without wealth effects the correlation of output and consumption is strongly positive, 0.90, as it is in the data. Finally, all labor market comovements are not substantially distorted with respect to our NCES-CD benchmark with a weak negative correlation of wages and output, slightly positive correlation of labor productivity and output, and a countercyclical labor share.

Overall, the results in this section suggest that incorporating adjustment costs and/or the use of GHH preferences into our NCES-CD model is useful for correcting the behavior of consumption, in particular, the negative correlation with output that was generated by investment and government shocks. Importantly, these two consumption-correction avenues do not distort our benchmark labor market results.

Table I-1: Robustness of Labor Market Results to Consumption Behavior

	Without V & G Shocks			With Adjustment Costs			With Habit Persistence			GHH Preferences		
	υ_x	$\rho(y,x)$	$\rho(x,x')$	υ_x	$\rho(y,x)$	$\rho(x,x')$	υ_x	$\rho(y,x)$	$\rho(x,x')$	υ_x	$\rho(y,x)$	$\rho(x,x')$
Output:												
y	2.24	1.00	0.72	3.34	1.00	0.87	3.17	1.00	0.72	3.40	1.00	0.82
Labor Market:												
eh	1.15	0.94	0.72	3.96	0.87	0.90	3.38	0.90	0.70	3.98	0.90	0.84
e	1.10	0.92	0.72	7.99	0.84	0.95	3.05	0.90	0.70	-	-	-
h	0.02	0.34	0.74	0.99	-0.64	0.98	0.03	0.55	0.72	-	-	-
w	0.11	0.57	0.84	1.04	-0.03	0.83	0.78	-0.18	0.75	1.04	-0.19	0.82
lp	0.38	0.80	0.74	0.94	0.09	0.80	0.64	0.14	0.70	0.77	0.06	0.79
ls	0.23	-0.63	0.72	0.27	-0.22	0.90	0.53	-0.37	0.80	0.57	-0.33	0.83
Elasticity:												
w/r	4.55	-0.88	0.71	7.14	-0.77	0.90	9.98	-0.76	0.73	11.96	-0.69	0.84
eh/k	1.29	0.88	0.73	4.02	0.87	0.90	3.56	0.87	0.71	4.08	0.88	0.84
σ	2.21	-0.56	0.90	4.85	-0.33	0.95	2.21	-0.46	0.90	2.64	-0.39	0.89
Cons./Inv.:												
c	0.06	0.50	0.91	1.60	0.68	0.87	0.06	-0.09	0.96	1.70	0.90	0.82
i	11.77	1.00	0.71	8.03	0.84	0.92	15.60	0.87	0.73	12.10	0.80	0.79
R	0.00	0.93	0.71	0.00	0.89	0.87	0.00	0.84	0.72	0.00	0.85	0.81
Shocks:												
A^*	1.42	0.96	0.71	1.39	0.58	0.77	1.42	0.76	0.71	1.15	0.60	0.76
V^*	-	-	-	1.04	0.14	0.94	0.98	0.25	0.92	1.04	0.11	0.94
G^*	-	-	-	0.84	0.42	0.71	0.84	0.41	0.71	0.84	-0.01	0.71
B^*	0.17	-0.12	0.74	17.26	-0.69	0.98	0.18	-0.11	0.75	2.01	-0.79	0.84
TFP Residual:												
Z^*	0.74	0.96	0.71	0.89	0.72	0.78	0.74	0.80	0.71	0.77	0.75	0.77

Notes: The set of columns 'Without V & G Shocks' removes investment and government shocks from the benchmark NCES-CD model; the set of columns 'With Adjustment Costs' adds these adjustment costs to the NCES-CD model; the set of columns 'With Habit Persistence' incorporates preferences with habit as in Boldrin et al. (2001); and the set of columns with 'GHH preferences' refers to the use of the no-wealth-effect preferences postulated in Greenwood et al. (1988) on our NCES-CD model. See Appendix I for further details.

J Re-Estimation of CES-CD and CD-CD Models

In Section 6.1.2 we compared our model results to the CES-CD and CD-CD models without re-estimating the model parameters for the CES-CD or CD-CD cases. Here we conduct the same exercise with re-estimation. Table J-1 shows the esitmation results. Table J-2 shows the unconditional business cycle moments for our benchmark NCES-CD model and the re-estimated CES-CD and CD-CD models. Table J-3 shows the associated labor market co-movements. Finally, Figure J-1 shows the IRFs of productivity, labor share and output in response to productivity shocks. Overall, we do not find substantial differences between our results with re-estimation and our results without re-estimation in Section 6.1.2 except for a now targeted, hence, lower volatility of the CD-CD model that matches the data.

Table J-1: Estimation Results [Simulated Methods to Match IRFs and Additional BC Moments]

	NCES(SR)-CD(LR)		CES(SR)-CD(LR)		CD(SR)-CD(LR)	
Parameters	Estimated	95% C.I.	Estimated	95% C.I.	Estimated	95% C.I.
σ	0.5981	[0.5943,0.6019]	0.4445	[0.4439,0.4452]	1.0000	-
ψ_{1a}	0.9536	[0.9518,0.9554]	0.9547	[0.9537,0.9556]	0.9543	[0.9036,1.0049]
ψ_{2a}	-0.0595	[-0.0618,-0.0571]	-0.0686	[-0.0698,-0.0675]	-0.0610	[-0.1174,-0.0046]
υ_a	0.0090	[0.0088,0.0092]	0.0096	[0.0096,0.0097]	0.0056	[0.0054,0.0058]
$\psi_{1\sigma}$	0.9830	[0.0394,1.9265]	-	-	-	-
$\psi_{2\sigma}$	-0.2049	[-0.7763,0.3666]	-	-	-	-
υ_σ	0.0145	[-0.0208,0.0499]	-	-	-	-
$\psi_{1a\sigma}$	-6.8144	[-8.2596,-5.3691]	-	-	-	-
$\psi_{2a\sigma}$	2.9812	[-2.4067,8.3691]	-	-	-	-
$\upsilon_{a\sigma}$	-0.4827	[-1.4951,0.5298]	-	-	-	-
ψ_{1b}	1.0370	[-0.2809,2.3550]	1.0166	[0.8288,1.2044]	0.9872	[0.3934,1.5810]
ψ_{2b}	-0.0619	[-1.2392,1.1153]	-0.1084	[-0.4000,0.1831]	-0.0114	[-0.0279,0.0050]
υ_b	0.0031	[-0.0003,0.0065]	0.0018	[0.0016,0.0021]	0.0037	[0.0013,0.0060]
ψ_{1v}	1.6763	[1.6762,1.6763]	1.6950	[1.6950,1.6951]	1.6772	[1.6770,1.6774]
ψ_{2v}	-0.6808	[-0.6809,-0.6808]	-0.7039	[-0.7040,-0.7038]	-0.6810	[-0.6812,-0.6809]
υ_v	0.0035	[0.0034,0.0036]	0.0060	[0.0059,0.0060]	0.0035	[0.0031,0.0039]
ν_2	-5.5979	[-6.6821,-4.5138]	-6.4956	[-6.7785,-6.2127]	-5.6998	[-12.0544,0.6548]
κ_2	1.9096	[1.8738,1.9455]	1.9407	[1.9319,1.9496]	1.9092	[1.6318,2.1867]
ζ	0.1192	[0.0926,0.1457]	0.1252	[0.1238,0.1265]	0.1764	[0.1696,0.1833]

Table J-2: The Quantitative Importance of σ_t (I) (w/ re-estimation)

	NCES(SR)-CD(LR)			CES(SR)-CD(LR) σ =.44 $\forall\, t$			CD(SR)-CD(LR) σ =1 $\forall\, t$		
	v_x	$\rho(y,x)$	$\rho(x,x')$	v_x	$\rho(y,x)$	$\rho(x,x')$	v_x	$\rho(y,x)$	$\rho(x,x')$
Output:									
y	3.28	1.00	0.72	3.34	1.00	0.76	3.28	1.00	0.77
Labor Market:									
eh	3.32	0.91	0.72	3.23	0.88	0.78	4.49	0.97	0.79
e	3.04	0.91	0.72	2.93	0.88	0.78	4.12	0.96	0.79
h	0.03	0.51	0.73	0.02	0.64	0.77	0.03	0.54	0.76
w	0.63	-0.14	0.75	1.14	-0.53	0.77	0.34	-0.41	0.80
lp	0.58	0.19	0.70	0.80	0.28	0.73	0.34	-0.41	0.80
ls	0.52	-0.35	0.80	0.76	-0.93	0.73	0.00	0.00	0.00
Elasticity:									
w/r	9.32	-0.77	0.73	18.51	-0.91	0.75	4.85	-0.92	0.80
eh/k	3.50	0.88	0.72	3.53	0.80	0.79	4.88	0.92	0.80
σ	2.21	-0.48	0.90	-	-	-	-	-	-
Cons./Inv.:									
c	0.72	-0.26	0.73	1.38	-0.58	0.77	0.48	-0.54	0.81
i	18.24	0.92	0.72	24.94	0.92	0.77	19.39	0.91	0.80
R	0.00	0.84	0.72	0.00	0.93	0.74	0.00	0.96	0.78
Shocks:									
A^*	1.42	0.79	0.71	1.63	0.71	0.71	0.54	0.68	0.71
V^*	0.98	0.24	0.92	3.48	0.45	0.93	1.03	0.51	0.92
G^*	0.84	0.34	0.71	0.81	0.36	0.71	0.81	0.41	0.71
B^*	0.18	-0.12	0.75	0.07	-0.05	0.74	0.23	-0.16	0.73
TFP Residual:									
Z^*	0.74	0.83	0.71	0.92	0.81	0.71	0.31	0.81	0.72

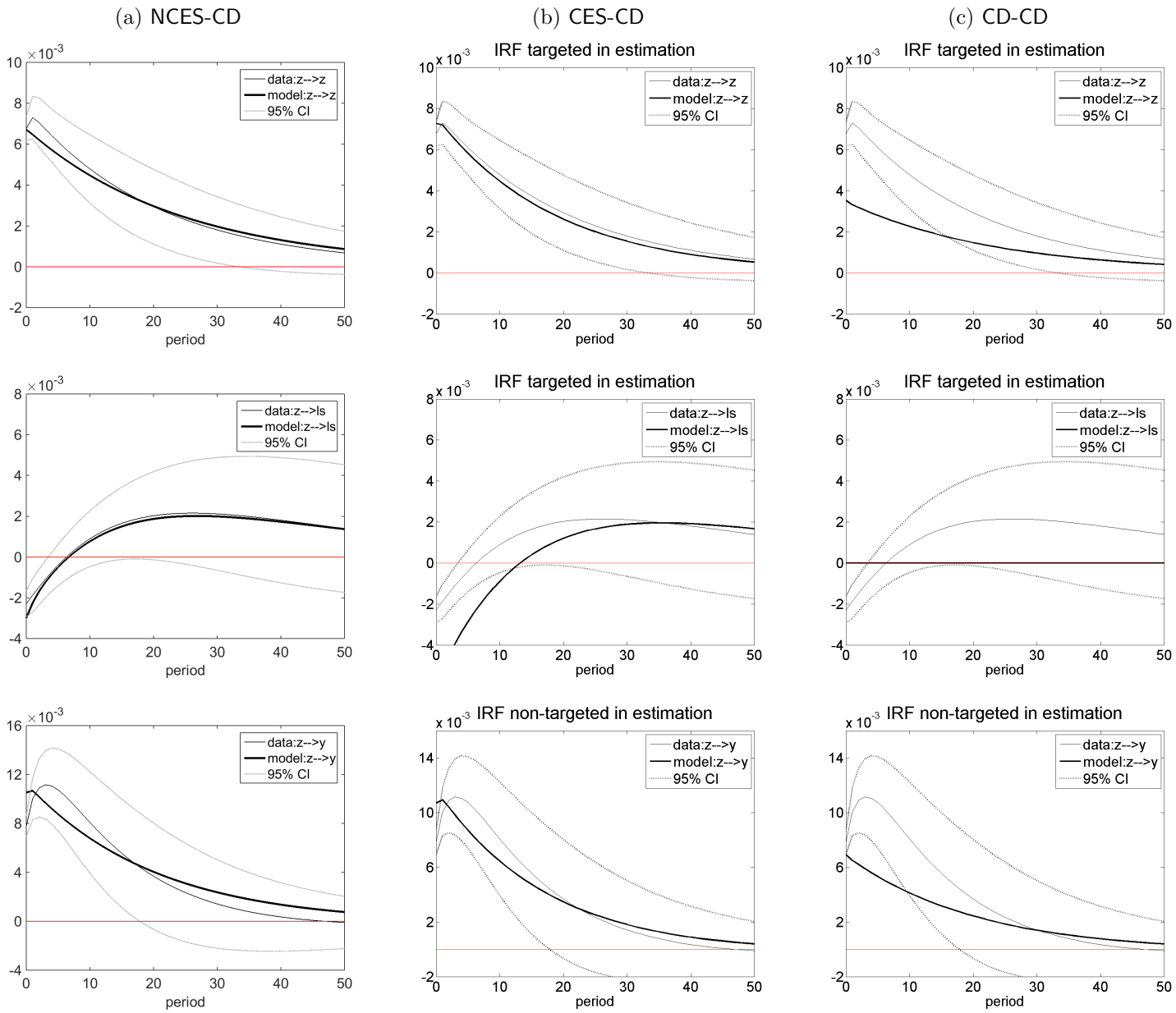
Notes: See Notes of Table 4.

Table J-3: The Quantitative Importance of σ_t (II): Labor Market Comovements (w/ re-estimation)

	y	eh	e	h	w	lp	ls
NCES(SR)-CD(LR) Model							
y	1.00	0.91	0.91	0.51	-0.14	0.19	-0.35
eh		1.00	1.00	0.54	-0.37	-0.23	-0.17
e			1.00	0.47	-0.38	-0.23	-0.17
h				1.00	-0.12	-0.07	-0.06
w					1.00	0.57	0.50
lp						1.00	-0.43
ls							1.00
CES(SR)-CD(LR) Model: σ =.44 $\forall\, t$							
y	1.00	0.88	0.88	0.64	-0.53	0.28	-0.93
eh		1.00	1.00	0.71	-0.85	-0.22	-0.81
e			1.00	0.68	-0.85	-0.22	-0.81
h				1.00	-0.58	-0.13	-0.58
w					1.00	0.62	0.59
lp						1.00	-0.27
ls							1.00
CD(SR)-CD(LR) Model: σ =1 $\forall\, t$							
y	1.00	0.97	0.96	0.54	-0.41	-0.41	0.00
eh		1.00	1.00	0.53	-0.63	-0.63	0.00
e			1.00	0.46	-0.63	-0.63	0.00
h				1.00	-0.24	-0.24	0.00
w					1.00	1.00	0.00
lp						1.00	0.00
ls							1.00

Notes: See Notes of Table 4.

Figure J-1: IRFs of Labor Share and Output to Z-shock (w/ re-estimation): NCES-CD vs. CES-CD vs. CD-CD



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