Quantitative Macroeconomics
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Homework 3, due April 19

Question 1. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{1}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{2}$$

$$y_t = k_t^{1-\theta} \ (z\ h_t)^{\theta} \tag{3}$$

$$i_t = k_{t+1} - (1-\delta)\ k_t \tag{4}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

1. Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.
2. Double permanently the productiviy parameter z and solve for the new steady state.
3. Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labor and output.

Question 2. Unexpected Shocks

Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.

Bonus Question: Can taxes explain differences in the speed of transition to steady-state?

1. Add a permanent consumption tax. Recompute the new steady state, and the transitions.
2. Add a permanent capital tax. Recompute the new steady state, and the transitions.

Bonus Question: Boldrin, Christiano and Fisher (AER, 2001) and Christiano (Minn QR, 1989)

1. What if preferences take the form of Boldrin, Christiano and Fisher (AER, 2001)? That is, abstracting from labor choice,
$$u(c) = \ln(c_t - bc_{t-1}). \tag{5}$$
Recompute the transition as posed in Question 1.

2. What if preferences take the form of Christiano (Minn QR, 1989)? That is, abstracting from growth,
$$u(c) = \ln(c_t - \bar{c}) \tag{6}$$
Recompute the transition as posed in Question 1. Plot the differences in the time path of savings.

3. Now, allow for growth, i.e., $z_t = z_0(1+\lambda_z)^t$, and replicate Christiano's Chart 1-4 for Japan, and extend the exercise to as many countries as you can (e.g. China, Taiwan, Korea, South Africa and Zambia). Get historical data for the U.K. (as long time series as you can), and replicate those Charts.

Bonus Question: Labor Choice

Allow for elastic labor supply. That is, let preferences be
$$u(c_t, 1-h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{7}$$
and recompute the transition as posed in Question 1.