Policy Evaluation

Quantitative Macroeconomics

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The biggest risk is not taking any risk... In a world that is changing really quickly, the only strategy that is guaranteed to fail is not taking risks." Mark Zuckerberg.

These notes describe the the welfare analysis suggested in Lucas (1987,2003) and some of its implementations, from the analysis of the welfare costs of business cycles to the analsys of the welfare gains from growth in an environment where more growth is associated with more income risk and less consumption insurance as it is the case of China described in work with Yu Zheng.

Preliminaries: Expectations of nonlinear functions and log-normal random variables

- Consider consumption is a random variable C with probability density function f. Consider another function g. Then,

$$E[g(C)] = \int g(c)f(c)dc \tag{1}$$

  Unless g is linear, in general:

$$E[g(C)] \neq g[E(C)] \tag{2}$$

  If g is convex, then $E[g(C)] \geq g(E[C])$. This is Jensen's Inequality.

- Let's assume that $X \sim N(\mu, \sigma^2)$. This normality assumption implies that the probability density function of X is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{3}$$

- **Theorem 1**: If $C \sim N(0, \sigma^2)$ then $E\left[\mathrm{e}^C\right] = \mathrm{e}^{\sigma^2/2}$

  Proof: Note that

$$E\left[\mathrm{e}^C\right] = \int_{-\infty}^{\infty} \mathrm{e}^c \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{c^2}{2\sigma^2}} dc \tag{4}$$

  Add and subtract $\sigma^2/2$ to the exponent (this is known as completing the square) to get (after rearranging):

$$E\left[\mathrm{e}^C\right] = \mathrm{e}^{\sigma^2/2} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\sigma^2/2 + c - \frac{c^2}{2\sigma^2}} dc = \mathrm{e}^{\sigma^2/2} \left[\int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{e}^{-\frac{\left(\sigma^2 - c^2\right)^2}{2\sigma^2}} dc\right] \tag{5}$$

  where note that the term in brackets is just the integral on the density of a variable that follows a normal with the mean equal to its variance, $N \sim (\sigma^2, \sigma^2)$. That is, the term in brackets is equal to one. QED.

- **Corollary:** If $C \sim N(\mu, \sigma^2)$, then $E[\mathrm{e}^C] = \mathrm{e}^{\mu + \sigma^2/2}$.

- We say that C is log-normally distributed if $\ln C \sim N(\mu, \sigma^2)$.[1]

- **Theorem 2**: If $\ln C \sim N(0, \sigma^2)$ then $E\left[C\right] = \mathrm{e}^{\sigma^2/2}$

  Proof: Write $C = e^{\ln C}$, where $\ln C \sim N(0, \sigma^2)$ and use Corollary.

[1] Which we sometimes write as $C \sim LN(\mu, \sigma^2)$ by convention.

- Suppose that we want to compare two scenarios. Say these scenarios are A and B. Assume one agent and one good for now.
- These scenarios can represent, for example:
  - A change in policy.
  - A world with high vs. low growth.
  - A world with high vs. low risk.
  - Rural vs. Urban.
- With scenario A consumer's welfare is $W(c_A)$, where c_A is consumption level under scenario A, and $W(c_b)$ is the analogous for scenario B.

- Let's assume that our agent prefers B, that is, $W(c_A) < W(c_B)$.
- Then, one way to measure welfare effects between these two scenarios is to find λ in

$$W((1+\lambda)c_A) = W(c_B), \tag{6}$$

  where λ is a constant—in untis of a percentage of the consumption good—that represents the *welfare gain* (loss) of a change in policy from A to B. In other terms, λ represents consumption compensations for variations across different scenarios (scenarios given by growth and/or risk

- Then, to evaluate the effects of policy we can find simply λ. A positive value indicates a *welfare gain*, that is, our agent's welfare would have to increase if it were moved from A to B and, hence, in order to remain indifferent at A our agent needs to be compensated by some $\lambda > 0$.

  That is, λ is the amount of consumption (in units of percentage) across all periods and states that individuals living in a reference scenario (in our case A) will demand to remain indifferent between their current scenario and the counterfactual B.

What is the Welfare Cost of Growth and Risk?

- Ideally, we we would like to jointly evaluate stabilization polices that could reduce aggregate risk and insurance policies that diversify risk across the households.
- Let's consider a single consumer endowed with the stochastic consumption stream,

$$c_t = Z\mathrm{e}^{gt}\mathrm{e}^{-\sigma^2/2}\varepsilon_t, \tag{7}$$

  where $\ln \varepsilon_t \sim N(0, \sigma^2)$.
- Note that under these assumptions,

$$E\left[\mathrm{e}^{-\sigma^2/2}\varepsilon\right] = 1 \tag{8}$$

  and mean consumption at t is $Z\mathrm{e}^{gt}$ and g is a deterministic growth rate.

- Preferences are assumed to be,

$$E\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}\frac{c_t^{1-\gamma}}{1-\gamma}\right], \tag{9}$$

  where $\beta = \frac{1}{1+\rho}$ is the discount factor, and γ is the coefficient of relative risk aversion.[2] The expectation is taken with respect to the common distribution of shocks ε_t.

- Our risk averse consumer will prefer a deterministic path to a risky path with the same mean.

[2] In this case, the inverse of γ is also the intertemporal elasticity of substitution.

- How can we quantify the welfare cost of risk? A way to do so is by finding λ in

$$E\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}\frac{\left((1+\lambda)c_t^{GR}\right)^{1-\gamma}}{1-\gamma}\right]=\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^{t}\frac{\left(c_t^{G}\right)^{1-\gamma}}{1-\gamma} \tag{10}$$

where we denote c_t^{GR} as our reference consumption path with growth and risk, and c_t^{GNR} is the deterministic consumption path with growth and no risk.

We quantify this utility difference (i.e., the welfare gain or cost) with the λ at all dates and states that makes the household indifferent between living in a world with growth and risk (GR) and a world with growth and no risk (GNR).

- How can we quantify the welfare cost of growth? A way to do so is by finding λ in

$$E\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \frac{\left((1+\lambda)c_t^{GR}\right)^{1-\gamma}}{1-\gamma}\right]=\sum_{t=0}^{\infty}\left(\frac{1}{1+\rho}\right)^t \frac{\left(c_t^{NGR}\right)^{1-\gamma}}{1-\gamma} \tag{11}$$

  where c_t^{NGR} is the consumption path associated with no growth and only risk. We can further evalute the case of a stagnant period with no growth and also no risk by changing the consumption path in the right hand side to c_t^{NGNR}.

Welfare Analysis

- In our courses the PILCH models (i.e., Ayagari-Bewley-Hugget economies) were stationary: The distribution of wealth (hence consumption) was invariant to time.
  - ▷ A stationary distribution of wealth implies aggregate output, investment, and consumption is also invariant to time.
  - ▷ That is, the model was silent about the how aggregate variables and distributions evolve over time (either with growth or over the business cycle).
- Here, we introduce one form of nonstationary economy:
  - ▷ The economy is not stationary along the transition to a stationary economy. This transition is generated by an unexpected shock to the environment of the economy (i.e., a policy change, demographic changes, an aggregate productivity shock, etc.)
  - ▷ There are other forms of nonstationary economies such as those derived from aggregate productivity shocks that hit the economy at every period. This is the Krusell and Smith (1998) economy. We will ignore these type of economies in these slides (check the Quantitative Macro course).

The Problem: Unexpected Policy Change and Transitional Dynamics

1. Suppose the economy is in a stationary equilibrium, given a government policy, preferences, endowments (labor earnings process) and technology.

2. Suppose there is an unexpected change (a zero probability event) in one of the exogenous elements in the model: government policy.

3. We want to study the transition path induced by the exogenous change, from the old stationary equilibrium to the new one.

- For instance, suppose an unexpected permanent introduction of a capital income tax at rate τ. The receipts are rebated lump-sum to households as government transfers, T.
- The initial policy is characterized by $\tau = T = 0$.
- Individuals are going to change the savings behavior and there will be a nontrivial transition path induced by the reform.

- Since the transition path is characterized by a sequence of prices, quantities and distributions we will cast the definition and solution of the model in sequential notation—the household problem still in recursive formulation.

- Let $Z = Y \times R_+$ be the set of all possible (y_t, a_t).

- Let $\mathcal{B}(R_+)$ be the Borel σ-algebra of R_+ and $\mathcal{P}(Y)$ the power set of Y.

- Let $\mathcal{B}(Z) = \mathcal{P}(Y) \times \mathcal{B}(R_+)$ and M be the set of all finite measures on the measurable space $(Z, \mathcal{B}(Z))$.

- The household problem is,

$$v_t(a, y) = \max_{c \geq 0, a' \geq 0} u(c) + \beta \sum_{y' \in Y} \pi(y'|y) v_{t+1}(a', y') \tag{12}$$

subject to

$$c + a' = w_t y + (1 + (1 - \tau_t) r_t) a + T_t \tag{13}$$

- The value functions are now functions of time because aggregate prices and polices change over time.

Definition (The competitive equilibrium)

Given the initial distribution Φ_0 and a fiscal legislation $\{\tau_t\}_{t=0}^{\infty}$, a competitive equilibrium is a sequence of individual household functions $\{v_t, c_t, a_{t+1} : Z \times M \to R\}_{t=0}^{\infty}$, sequence of production plans $\{N_t, K_t\}_{t=0}^{\infty}$, factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, government transfers $\{T_t\}_{t=0}^{\infty}$ and a sequence of measures $\{\Phi\}_{t=1}^{\infty}$ such that, $\forall t$,

1. Given $\{w_t, r_t\}$ and $\{T_t, \tau_t\}$ the functions v_t solve Bellman's equation for period t and c_t, a_{t+1} are the associated policy functions.
2. Factor prices $\{w_t, r_t\}$ satisfy $w_t = F_L(K_t, L_t)$ and $r_t = F_K(K_t, L_t) - \delta$.
3. Balanced Government Budget: $T_t = \tau_t \; r_t \; K_t$.
4. Market Clearing

$$\begin{aligned} \int c_t(y_t, a_t) d\Phi_t + K_{t+1} &= F(K_t, L_t) + (1-\delta)K_t \\ L_t &= \int y_t d\Phi_t \\ K_{t+1} &= \int a_{t+1}(y_t, a_t) d\Phi_t \end{aligned}$$

5. Aggregate Law of Motion: $\Phi_{t+1} = \Gamma_t \Phi_t$.

- **A Stationary Equilibrium** is an equilibrium such that all elements of the equilibrium that are indexed by t are constant over time.

Computation of the Equilibrium Transition Path

- What we are after:

  1. At $t = 0$ we have a stationary equilibirum with τ_0 and associated equilibrium distribution Φ_0 (hence we have K_0, r_0, w_0) and associated value function v_0 and decision rules c_0, a_1.

  2. At $t = 1$ policy changes permanently to $\tau_t = \tau > 0$ for all $t \geq 1$.

  3. Denote the new stationary equilibrium associated with τ by Φ_∞ with associated value function v_∞ and decision rules c_∞, a_∞.

- We want to compute the entire transition path and compute the welfare consequences of such policy innovation.

- How can we compute the transition path?
  - ▷ Assume it takes T periods to move from the old stationary equilibrium to the new one.
  - ▷ T should be sufficiently large so that the new stationary is reached.
  - ▷ Using the fact that $v_T = v_\infty$, then for a given sequence of prices $\{r_t, w_t\}_{t=1}^{T}$ the household problem can be solved backwards. This is independent of whether people leave forever or not!

Algorithm (Computing the transitional dynamics)

1. *Fix T.*
2. *Compute stationary equilibrium at $t = 0$.*
3. *Compute stationary equilibrium at $t = \infty$ assuming that stationarity is reached at $t = T$.*
4. *Guess a sequence of prices and transfers choosing $\{\hat{K}_t\}_t^{T-1}$ (note that $\hat{K}_1 = K_0$ and $L_t = L_0 = \bar{L}$ is fixed):*
$$\hat{w}_t = F_L(\hat{K}_t, \bar{L}),\ \hat{r}_t = F_K(\hat{K}_t, \bar{L}), \text{ and } \hat{T}_t = \tau_t \hat{r}_t \hat{K}_t.$$
5. *Since we know $v_T(a, y)$ and $\{\hat{r}_t, \hat{w}_t, \hat{T}_t\}_{t=1}^{T-1}$ we can solve for $\{\hat{v}_t, \hat{c}_t, \hat{a}_{t+1}\}_{t=1}^{T-1}$ backwards.*
6. *With the sequence for $\hat{a}_{t+1}$ we can define the transition laws $\{\hat{\Gamma}_t\}_{t=1}^{T-1}$. Since we know that $\Phi_1 = \Phi_0$ from the initial stationary equilibrium, we can iterate the distributions forward*
$$\hat{\Phi}_{t+1} = \hat{\Gamma}_t \hat{\Phi}_t$$
7. *With $\{\hat{\Phi}_t\}_{t=1}^{T}$ we can compute*
$$\hat{A}_t = \int a \, d\hat{\Phi}_t$$
8. *Check whether*
$$\max_{1 \leq t < T} |\hat{A}_t - \hat{K}_t| < \varepsilon$$
If yes, go to 9. If not, adjust your guesses for $\{\hat{K}_t\}_{t=1}^{T-1}$ in step 4.
9. *Check whether $|\hat{A}_T - \hat{K}_T| < \varepsilon$. If yes, we are done. If not, go back to step 1 and adjust T.*

- It turns out that with the sequence of value functions v_t we can make statements about welfare. What are the welfare consequences of a tax reform?

Measuring Welfare Consequences of Policy

To measure the welfare consequences of unexpected policy change we need to take into account the entire transtion path.

- The welfare consequences are a result of interpreting value functions:
  - ▷ Function $v_0(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 0 in the initial stationary equilibrium—i.e., for a person that thinks he will live in the stationary equilibrium with $\tau = 0$ forever.
  - ▷ Function $v_1(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 1 that has just been informed that there is a permanent tax change—i.e., this $v_1(a, y)$ takes into account all the transition dynamics through which the agent is going to live.
  - ▷ Function $v_T(a, y) = v_\infty(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 0 in the final stationary equilibrium—i.e., this agent does not live during the transition.

- In principle, then, we can use v_0, v_1 and v_T to assess welfare consequences of reforms.
- But utility is an ordinal concept that we cannot quantify.

- To get around we can compute a consumption equivalent variation. To do so consider the optimal consumption allocation in the initial stationary equilibrium $\{c_s\}_{s=0}^{\infty}$, a CRRA utility and the associated $v_0(a, y)$ is

$$v_0(a, y) = E_0 \sum_{s=0}^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Suppose we increase consumption in each date by a fraction g so that the new allocation is $\{(1+g)c_s\}_{s=0}^{\infty}$. The lifetime utility from that consumption allocation is:

$$\begin{aligned} v_0(a, y; g) &= E_0 \sum_{s=0}^{\infty} \frac{[(1+g)c_s]^{1-\sigma}}{1-\sigma} \\ &= (1+g)^{1-\sigma} E_0 \sum_{s=0}^{\infty} \frac{c_s^{1-\sigma}}{1-\sigma} \\ &= (1+g)^{1-\sigma} v_0(a, y) \end{aligned}$$

where

$$v_0(a, y; g = 0) = v_0(a, y).$$

- If we want to quantify the welfare consequences of the policy reform for agent (a, y) we can ask: by what percent g do we have to increase consumption in the old stationary equilibrium in each date and state for the agent to be indifferent between living in the old stationary equilibrium and living through the transition induced by the policy reform.

- This percent g solves

$$v_0(a, y; g) = v_1(a, y)$$

or, rearranging:

$$\begin{aligned} (1+g)^{1-\sigma} v_0(a, y) &= v_1(a, y) \\ g(a, y) &= \left[\frac{v_1(a, y)}{v_0(a, y)}\right]^{\frac{1}{1-\sigma}} - 1 \end{aligned}$$

- If $g(a, y)$ is bigger than zero agents will benefit from the reform and $g(a, y)$ measures how much in consumption terms.

- Note that $g(a, y)$ depends on a and y. In the event of a tax increase on capital income one would expect households with a lot of assets lose badly, while households with little assets may even gain (taxes are lump-sum redistributed).

- Note that we only need to know $v_0(a, y)$ and $v_1(a, y)$ to compute $g(a, y)$, but our computation of the transition path gives us already v_1.

- Often studies ignore—with no argument for it: laziness is NOT an argument!—the transition and assess the steady state welfare consequences of policy reform.
  - ▷ To do so one computes

$$g_{ss}(a,y) = \left[\frac{v_T(a,y)}{v_0(a,y)}\right]^{\frac{1}{1-\sigma}}$$

- This is interpreted as steady-state welfare gain of an agent being born with characteristics (a, y).
- Steady state comparisons ignore the transition with possibly quantifiable consequences. For example, an increase in capital tax induces in principle a lower aggregate capital, hence consumption and a loss of welfare. However, along the transition path part of the capital stock is being eaten with the associated consumption and welfare derived from it.
- Whenever possible, avoid the parallel universe comparisons.

- We can also check the welfare consequences of a policy reform before households characteristics are revealed. At the steady state this is:
    - ▷ To do so one can compute

$$g_{ss} = \left[\frac{\int v_T(a,y) d\Phi_T}{\int v_0(a,y) d\Phi_0} \right]^{\frac{1}{1-\sigma}}$$

- We interpret this name as the welfare gain of an agent being born with characteristics (a, y) where $\int v_T(a,y)d\Phi_T$ is the expected lifetime utility of an agent in the new steady state, before the agent knows his pair (a, y)—behind the veil of ignorance (John Rawls). $\int v_0(a,y)d\Phi_0$ is defined accordingly.

Example I: Lucas 1987 and Lucas 2001: Aggregate Risk and Growth (no Individual Risk)

- **Main result:** The welfare gains of entirely removing business cycles (an extreme stabilization policy) are small, about $\lambda =.05$ for $\gamma =1$ (log-preferences) and $\lambda =.12$ for $\gamma =2.5$. That is, in order to remain in world $\{GR\}$ our agent needs to be compensated by an increase in consumption across all dates and sates of .05% if he has log-preferences, and by an increase of .12% if his preferences are defined by $\gamma =2.5$.
- The welfare costs of growth are much larger with λ about -30 if the counterfactual is c^{NGR}.
- Note that there is no individual risk in this exercise, that is, insurance policies across households are not evaluated.

Example II: Krusell and Smith 1998: Aggregate Risk and Individual Risk (no growth)

- They introduce household level risk (employment risk) into an otherwise standard business cycle model (i.e., a model with aggregate risk). These household level risk cannot be diversified with households subject to borrowing constraints. A big contribution is to provide of this paper is to provide an algorithm to solve for such economies.

- What is the effect of removing aggregate risk (also removing the effect that aggregate risk has on individual risk, i.e., in good times it s more likely you are unemployed)? The welfare gain is $\lambda = .0001$, i.e., .01% increase on average consumption. Low wealth, however, would get gains of 4%. Also rich guys gain, about 2% induced by the increase in the interest rate due to a decline in precautionary savings. There are still middle class individuals that suffer welfare losses (through the still presence of precautionary savings that increases their within group average consumption).
- Overall, KS findings are similar to Lucas'. The message has then been that individual insurance is not quantitatively important for welfare. To investigate policy, we might be fine with representative agent models. This result has been some times called to justify reprensetative agent models in development too.

Example III: Storesletten et al. 2001: Aggregate Risk and Individual Risk (no growth)

- They use an overlapping generation model with 43 working generations with the youngest being credit constrained always. For the young, then, all reductions in risk implies a welfare gain. But if the age effects are averaged out (i.e, reflecting for instance intrafamily lending) the gains are similar to KS.

- With $\gamma =$2, removing both aggregate and individual risk implies a gain of $\lambda =$.59 on average (of 1.50 for new borns), and with $\gamma =$4 the gains are 2.49% on average and 7.37% for new borns. They do the interesting decomposition of removing aggregate and individual risk. For the average guy the gains from removing aggregate risk are about 1.5 of those of removing individual risk with $\gamma =$2 and 4.
- Numbers have been revisited by many (see the works by Storesletten, Heathcote and Violante). New versions of Krusell-Smith with co-authors suggest welfare gains of 30% for the most vulnerable groups, but still low action on welfare gains on average.

Example IV: Santaeulalia-Llopis and Zheng 2016: Consumption Insurance vs. Economic Growth (China)

Findings:

1 Economic growth is associated with more income risk and a loss of consumption insurance. The transmission of permanent income shocks to consumption that triples from 1989 to 2009

2 Empirical evidence suggests that the shortage of available options to store wealth limits the use of household savings for precautionary reasons.

3 These results have implications for the welfare assessment of growth across time and space:

- Across time, the loss of insurance implies that rural households would actually prefer to live the growth-risk-insurance environment of the pre- rather than post-WTO years, despite higher rural growth in the 2000s.
- Across space, while the welfare gains of rural-to-urban migration substantially drop by more than two-thirds due to consumption insurance losses in pre-WTO years, these gains remain high in the post-WTO years after a change in the composition of public transfers that substantially improves insurance in urban areas compared with rural areas.

- China presents a clear trade-off between growth and insurance. Given the quantitatively large implications that we find insurance has for welfare in a growing economy, we argue that a good growth theory should incorporate insurance. We know that income per capita differences across countries are not necessarily welfare differences across countries, but we tend to ignore this discrepancy perhaps under the notion that insurance is not that relevant for welfare (this notion is perhaps guided by extrapolations of US results). Our exercise casts doubt on that practice. Do other developing countries present a similar growth vs. insurance trade-off?
- When one considers welfare, growth policies should be jointly determined with social insurance policies.