Quantitative Macroeconomics

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Lecture location and times: UAB Seminar Room, Monday & Wednesday 15.00-16.30

I Course Description

This course is Unit I (out of 2) of the second-year PhD quantitative macro sequence at UAB. The second unit will be taught by Luis Rojas. In this unit we will emphasize the use of projection methods to solve for the equilibrium allocations of heterogeneous agent models and overlapping generations models taking good care of distributions and aggregate consistency in stationary and non-stationary environments (e.g., business cycles or development processes). We will pose quantitative questions and learn how to answer them by doing. This will require intensive computational work by students.

You should check the course website regularly at:

http://r-santaeulalia.net/Quantitative-Macro-F18-UnitI.html.

There I will post announcements, the slides, homework sets, additional references if needed, and a class diary that keeps track of what we are doing.

II Grades and Requirements

This course is demanding and I expect you to be engaged continuously. The grade will be some weighted average of homework sets and presentations. In this course we are learning methods, and we learn them by using them. You should expect one mandatory homework per foreseeable week. One of you (not necessary at random) will present his/her homework solutions at the beginning of each class for 10-15 minutes.

You are definitely encouraged to work in groups but you will submit your homework sets individually: you will place the solution to the homeworks (and to possibly other requirements) in electronic form in some dropbox folder to be determined.

III Computer Skills

It helps if you have had previous programming experience but it is not a pre-requisite at all. However, in order to solve the homework sets it is definitely a requisite to learn how to program. To learn so, you are on your own. That is, I am not going to teach you how to declare variables, generate random numbers, call intrinsic functions, link external subroutines, etc., but I am assuming that you are gaining—for those who do not have it yet—expertise on programming by yourselves as we go along. In other words, this is not a computer science course; instead, what we learn in this class are tools and algorithms that, making use of some programming language, help us to solve modern macroeconomic models with heterogeneous agents.

The programming language you use is at your discretion. If you are planning to do serious computational work in your research, I encourage you to learn Fortran (good alternatives are C, C++ or Python). This requires an initial fixed cost but I think it pays off. When it comes to numerical work, the scientific community speaks Fortran and most large-scale scientific computer programs are written in Fortran. One good reason to do so is that Fortran is faster than other available alternatives, and you will care a lot about the speed when you increase the scale of your work—the second half of this course with heterogeneous agents will approach that boundary. Two good sources to learn how to program in Fortran are Chapman (1998) and http://www.cs.mtu.edu/~shene/COURSES/cs201/NOTES/fortran.html.

A must is https://lectures.quantecon.org/ by Thomas J. Sargent and John Stachurski. The provide lots of information on numerical methods and algorithms. They also provide code for Julia and Python with an interesting discussion of pros and cons.

Matlab is more user-friendly than Fortran and very popular in economics. It is particularly useful if you are used to think in vector-matrix operations. This application has a large amount of toolkits available to solve representative-agent business cycle models via (log- and) linearizations around steady states (see Uhlig's toolkit, Dynare, etc.) that you may find useful. Some alternatives are Gauss, R, Scilab and Octave.

To do serious data work when you are 'fishing for facts', Stata is a good application that allows you to upload and manipulate many large data sets at once. SAS or Eviews may work fine as well.

Finally, if time permits, we will learn the basics of parallel programming either through message passing interface or the use of GPUs.

IV Textbooks

A relevant set of references—in addition to standard macroeconomic textbooks such Stokey and Lucas (1989) and Sargent and Ljungqvist (2004)—that a graduate student that plans to use computational methods in his/her research should have is:

1. Cooley (1995)
2. Marimon and Scott (1998)
3. Judd (1998)
4. Press et al. (1992) (you can find this available and free online).
5. Heer and Maussner (2005)

These two recent handbook chapters Maliar and Maliar (2014) and Fernández-Villaverde et al. (2016) will quickly take you to the frontier. Students are also encouraged to checkon the lecture notes by Dirk Krüger (UPenn), and Gianluca Violante (Princeton). Though we will not strictly follow any of the previous references, you will see many obvious intersections between what we cover in class and the material presented in those books—I will try to refer you to the relevant, with respect to this course, parts of them. Also, note that References 4 and 5 provide code online.

V Course Outline

1. **Discrete Time Stochastic Dynamic Programming**

   (a) Finite Horizon and the Theorem of the Maximum

   (b) Infinite Horizon and the Contraction Mapping Theorem

   References: Harris (1987), Stokey and Lucas (1989), Sargent and Ljungqvist (2004), and Heer and Maussner (2005).

2. **Some Useful Numerical Methods**

   (a) Function Approximation (one- and multi-dimensional)
      - Local Methods: Taylor (and Padé)
      - Global Methods
        - Discretization
        - Spectral Methods: Polynomial Interpolation
          - Linear, kth-Order Polynomials, Chebyshev and other Orthogonal Basis
        - Finite Element Methods: Piecewise Polynomial Splines
          - Linear, Quadratic, Cubic Splines
          - Shape-Preserving Schumaker Splines
          - B-Splines
        - Weighted residuals methods: Collocation, Least Squares, Garlekin

   (b) Numerical Differentiation

   (c) Numerical Integration
      - Newton-Cotes Methods, Trapezoid Rule, Simpson's Rule
      - Gaussian quadrature
      - Monte Carlo Methods and Quasi-Monte Carlo Methods

   (d) Root Finding (solving systems of equations):
      - Bisection, Secant Method, Newton's Method, Fixed-Point Iteration
      - Gauss-Jacobi, Gauss-Seidel, Fixed-Point Iteration, Newton's Method, Secant (Broyden) Method, Enhancing Global Convergence: Powell's Hybrid, Homotopy Continuation Methods

   (e) Numerical Optimization

References: Marimon and Scott (1998), Judd (1998), Heer and Maussner (2005), and Press et al. (1992).

3. **Representative Agent Models**

   Neoclassical growth model with stand-in households, the workhorse of modern macroeconomics; real business cycle (RBC) models; and, additional cases with a large state space.

   (a) Value Function Methods
      - Value function iteration (VFI): From discretization to continuous methods.
      - Finite element methods and Collocation
      - Weighted residuals methods

   (b) Euler Equation Methods
      - Policy function iteration (PFI)
      - Finite element methods and Collocation
      - Weighted residuals methods

   References: Heer and Maussner (2005) and Aruoba et al. (2006).

4. **Heterogeneous Agent Models**

   (a) **Heterogeneous Agents with Complete Markets**
      i. When heterogenous agents economies behave as representative agent economies: The Negishi method
      ii. When heterogenous agents economies do NOT behave as representative agent economies.
      iii. The case of Hybrid OLGs Economies.

      References: Chatterjee (1994), Caselli and Ventura (2000), Maliar and Maliar (2001), Maliar and Maliar (2003) and Koulovatianos (2005).

   (b) **Heterogeneous Agents with Incomplete Markets: Aiyagar-Bewley-Hugget-Imrohoroglu Economies**
      i. Solution methods to infinitely lived economies
      ii. Solution methods to life-cycle economies
      iii. Solution methods to overlapping generations (OLGs) economies

      References: Imrohoroglu (1989), Huggett (1993), Aiyagari (1994), Castañeda et al. (2003), Conesa et al. (2009), and Krueger and Ludwig (2016)

References

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