Quantitative Macroeconomics

Raül Santaeulàlia-Llopis
MOVE-UAB and Barcelona GSE

Homework 5, due Mon Oct 28 at 8.30pm

Discuss all your results in each and all the following questions.

Question 1. Factor Input Misallocation

1. Firm-specific output, capital and productivity are, respectively, y_i, k_i and z_i. Assume that $\ln z_i$ and $\ln k_i$ follow a joint normal distribution. Assume that the correlation between $\ln z_i$ and $\ln k_i$ is zero, the variance of $\ln z_i$ is equal to 1.0, the variance of $\ln k_i$ is equal to 1.0, and that average s and k is equal to one. Then simulate 10,000,000 observations and plot the joint density in logs and in levels. We are going to assume that these 10,000,000 observations are your complete (or administrative) data that captures the entire population/universe of firms in a given country..

2. Compute firm output y_i for each of your observations as:

$$y_i = s_i^{1-\gamma} k_i^{\gamma}$$

   where $z_i = s_i^{1-\gamma}$ and $\gamma = 0.6$. From now on consider your simulated s_i, k_i and y_i as actual data.

3. Solve the following maximization problem:

$$Y^e = \max_{k_i} \sum_i s_i^{1-\gamma} k_i^{\gamma}$$

   subject to $K = \sum_i k_i$, where K is a parameter equal to the aggregate capital computed by adding up your complete data. To solve this problem use s_i from the actual data that you created in item 1.

4. Compare the optimal allocations k^e against the data.

5. Compute the ouptut gains from reallocation,

$$\left(\frac{Y^e}{Y^a} - 1 \right) * 100$$

   where $Y^a = \sum_i y_i$ is the aggregate actual output that you created in item 1 and 2.

6. Redo items (2)-(5) assuming that the correlation between $\ln z_i$ and $\ln k_i$ is 0.50. Redo with correlastion -0.50.

Question 2. Higher Span of Control

1. Redo the previous Question 1 for $\gamma = 0.8$. Discuss your results.

Question 3. From Complete Distributions to Random Samples

Assume now that the complete data (i.e., the actual population) that you generated in Question 1 is not accessible. You have a budget limit that allows you to interview only a sample of your complete data. This is almost invariably the case in survey data. Typically, surveys are designed to be collected using some form of (stratified or not) random sampling in order to satisfy representativeness of some characteristics of the entire population:

1. Please, random sample (without replacement) 10,000 observations. That is, your data sample implies a sample-to-population ratio of 1/1,000. What is the variance of $\ln z_i$ and $\ln k_i$ in your random sample? How do they compare compare to the complete data? How about the correlation between $\ln z_i$ and $\ln k_i$?

2. Redo items (3) to (5) in Question 1 for your random sample of 10,000 firms. Compare your results for misallocation using your random sample to the results obtained using the complete distribution.

3. Do the previous two items 1,000 times. Notice that each random sample is drawn from the entire population. This implies that you will compute 1,000 measures of misallocation. Show the histogram of the output gains, and provide some statistics of that distribution of these output gains, in particular, the median. Discuss your results.

4. What is the probability that a random sample delivers the misallocation gains within an interval of 10% with respect to the actual misallocation gains obtained from complete data?

5. Redo items (1)-(4) for three different sample-to-population ratios. In particular, use do the cases in which your random sample extracts 100 observations, 1,000 observations and 100,000 observations. That is, your sample implies a sample-to-population ratio of, respectively, 1/100,000, 1/10,000, and 1/100. Compare your results in items (1)-(4) to those obtained with the previous random sample size and the complete data.

,

Some Optional Questions:

[Optional] Question 1. Dynamic Misallocation

1. Write down a dynamic version of our probem with an exogenous barrier to capital accumulation. How would you solve the problem? How will the solution of this problem compare to its static version?

[Optional] Question 2. Selection

1. A firm now can operate either in sector a or sector na. Write down a version of our problem in which the choice in which sector to operate is endogenous and reassess the exent of misallocation.

[Optional] Question 3. Endogenous Productivity

Use the 1,000,000 observations from Question 1.1 to do the following items:

1. Compute firm output y_i for each of your observations as:

$$y_i = (s_i(a_i, k_i))^{1-\gamma} k_i^{\gamma}$$

where $\gamma = 0.5$. We assume that managerial ability now depends on the level of capital with come degree of complementerity between true ability (a_i) and capital (k_i),

$$s_i(a_i, k_i) = \left[\alpha a_i^{\frac{\sigma-1}{\sigma}} + (1-\alpha) k_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\alpha = 0.5$ and $\sigma = 1.0$. From now on consider your simulated s_i, k_i and y_i as actual data.

2. Solve the following maximization problem:

$$Y^e = \max_{k_i} \sum_i (s_i(a_i, k_i))^{1-\gamma} k_i^{\gamma}$$

subject to $K = \sum_i k_i$. To solve this problem use s_i from the actual data that you created in item 1.

3. Compare your the optimal allocations k_i^e against the data.

4. Compute the ouptut gains from reallocation from the data that you created in item 1.

5. Redo items (2)-(5) for $\sigma = 0.5$ and $\sigma = 2.0$. Discuss your results.

6. Compare your results to Question in this question to those in Question 1.