Quantitative Macroeconomics
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Homework 1, due Friday Sep 28

Question 1. Function Approximation: Univariate

1. Approximate $f(x) = x^{.321}$ with a Taylor series around $\overline{x} = 1$. Compare your approximation over the domain (0,4). Compare when you use up to $1, 2, 5$ and 20 order approximations. Discuss your results.

2. Approximate the *ramp function* $f(x) = \frac{x+|x|}{2}$ with a Taylor series around $\overline{x} = 2$. Compare your approximation over the domain (-2,6). Compare when you use up to $1, 2, 5$ and 20 order approximations. Discuss your results.

3. Approximate these three functions: $e^{\frac{1}{x}}$, the *runge* function $\frac{1}{1+25x^2}$, and the *ramp* function $f(x) = \frac{x+|x|}{2}$ for the domain $x \in [-1, 1]$ with:
   - Evenly spaced interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.
   - Chebyshev interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.
   - Chebyshev interpolation nodes and Chebyshev polynomial of order 3, 5 and 10. How does it compare to the previous results? Report your approximation and errors.

4. Approximate the following probability function:

$$p(x) = \frac{e^{-\alpha x}}{\rho_1 + \rho_2 e^{-\alpha x}}$$

   for the domain $x \in (0, 10]$ using Chebyshev interpolation nodes and Chebyshev polynomial of order 3, 5 and 10. Report your approximation and errors. Do this for two combinations of paramters $\alpha = 1.0$, $\rho_1 = \frac{1}{100}$ and $\rho_2 = \frac{1}{0.2}$. Redo for $\rho_2 = \frac{1}{0.25}$. Plot the results for both combinations in the same graph.

Question 2. Function Approximation: Multivariate

Consider the following CES function $f(k, h) = \left((1-\alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$ where σ is the elasticity of substitution (ES) between capital and labor and α is a relative input share parameter. Set $\alpha = 0.5$, $\sigma = 0.25$, $k \in [0, 10]$ and $h \in [0, 10]$. Do the following items:

- Show that σ is the ES (hint: show this analytically).
- Compute labor share for an economy with that CES production function assuming factor inputs face competitive markets (hint: compute this analytically as well).
- Approximate $f(k, h)$ using a 2-dimensional Chebyshev regression algorithm. Fix the number of nodes to be 20 and try Cheby polynomials that go from degree 3 to 15. For each case, plot the exact function and the approximation (vertical axis) in the (k, h) space.
- Plot the exact isoquants associated with the percentiles 5, 10, 25, 50, 75, 90 and 95 of output. Use your approximation to plot the isoquants of the your approximation. Plot the associated errors per each of these isoquant.
- For each case, show the associated approximation errors (vertical axis) in the (k, h) space.
- Redo using $\sigma = 5.00$ and $\sigma = 1.00$