Quantitative Macroeconomics

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Homework 4, due Tue Oct 10

Question 1. Sparse matrix manipulation

Consider a model economy with a Markovian income process that has as generic element in the transition matrix

$$\pi_{i,j} = Prob\left[y_{t+1} = y_j | y_t = y_i\right],$$

where:

- there are n =1001 possible income states;
- if 10$\geq i$, then $\pi_{i,0}$ =.34, and $\pi_{i,i} = \pi_{i,i+10}$ =.33.
- if 990$\geq i$ >10, $\pi_{i,i-10} = \pi_{i,i} = \pi_{i,i+10}$ =.33.
- if i >990, $\pi_{i,n-10} = \pi_{i,i} = \pi_{i,n}$ =.33.
- also, in each state i >10, there is, a small probability, .01, of earning the lowest income, y_0, next period.

1. Compute the invariant distribution of income of this economy.
2. Compute the mean, variance, skewness, Gini and coefficient of variation of the invariant distribution.
3. Use as initial distribution the current U.S. income distribution, and apply to it the transition matrix of the model economy iterating forward until the invariant distribution is reached.
   (a) Report again the invariant distribution of this economy, and its invariant statistics (do they change with respect to your previous computations?)
   (b) How many periods does it take for the U.S. income distribution to transit to the model invariant distribution?
   (c) Report the evolution of income inequality, the Gini, over the transition to the stationary distribution.
4. Modify the Markovian transition matrix to increase income mobility, and redo Question 3 and highlight the differences.