Quantitative Macroeconomics
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Question 1. A Dynamic Model of Occupational Choice

There is a unit measure of agents heterogeneous in their permanent productivity as workers, y, permanent productivity as entrepreneurs, z, and in their initial wealth, ω. Suppose the triplet (y, z, ω) is distributed as $G(y, z, \omega)$ in the population. Agents have a time-separable CRRA utility function with coefficient σ and discount factor β. There are two periods. At the beginning of the first period, an agent indexed by (y, z, ω) chooses his occupation for the next period, whether to be a worker or to become an entrepreneur.

- If an agent chooses to become a worker then in the first period he saves (or borrows) at the risk-free interest rate r and consumes, and in the second period he earns y' and earns the return on savings (or pay the debt), and consumes. The labor income in period two, y' is the sum of a permanent component, y, and a transitory (risk) component ε_y with $E(\varepsilon_y) = 0$. With probability one half, he receives a positive shock ε_y, and with the complementary probability, he receives a negative shock $-\varepsilon_y$. He solves the following problem, taking the interest rate r as given:

$$V^w(\omega; y, \varepsilon_y) = \max_{a,c} \quad u(c) + \beta E u(c')$$
$$\text{subject to } c + a = \omega$$
$$c' = y + \varepsilon_y + ra$$
$$a \geq -\frac{1}{r}(y - \varepsilon_y).$$

  where the last inequality is the natural borrowing constraint. It ensures that we can pay back our debt (even in the event of the worst shock occuring). Note that $Eu(c')$ is simply equal to $\sum_{\varepsilon_y} Prob(\varepsilon_y) u(c')$.[1]

  Note that by plugging the per-period budget constraints into the objective function, and spelling out the properties of the shock, we can write our problem as one in which we simply need to pin down a in:

$$V^w(\omega; y, \varepsilon_y) = \max_{a} \quad u(\omega - a) + \frac{1}{2}\beta \left[u(y + \varepsilon_y + ra) + u(y - \varepsilon_y + ra)\right]$$
$$\text{subject to } a \geq -\frac{1}{r}(y - \varepsilon_y).$$

- If the agent chooses to become an entrepreneur, then in the first period he saves (or borrows) at the risk-free rate r, invests in physical capital k and consumes, and in the second period he produces using z and k, sells the output, earns the return on savings (or pay the debt),

[1] Note that the earnings processes that we usually estimate from data takes the log form, i.e., $\ln y e^{u_y} = \ln y + u_y$. Then, it is important to note that we can rewrite $y + \varepsilon_y = y e^{u_y}$, that is, $\varepsilon_y = y(e^{u_y} - 1)$.

Parameter	Value
β	0.98
α	1/3
σ	2
y	[0.8,1.0,1.2,1.4,1.6]
z	[0.6,1.0,1.40,1.80,2.20]
ε_y	0.2
ε_z	0.5
r	0.04

Notes: That is, there are five number of ability types per occupational sector.

and consumes. The entrepreneurial output is likewise risky, with the size of the shock being ε_z. He solves the following problem taking the interest rate r as given:

$$\begin{aligned} V^e(\omega; z, \varepsilon_z) = \max_{a,c} \quad & u(c) + \beta E u(c') \\ \text{subject to } & c + a + k = \omega \\ & c' = (z + \varepsilon_z)k^\alpha + ra \\ & a \geq -\frac{1}{r}(z + \varepsilon_z)k^\alpha. \end{aligned}$$

That is,

$$\begin{aligned} V^e(\omega; z, \varepsilon_z) \max_{a,k} \quad & u(\omega - a - k) + \frac{1}{2}\beta \left[u((z + \varepsilon_z)k^\alpha + ra) + u((z - \varepsilon_z)k^\alpha + ra) \right] \qquad (1) \\ \text{subject to } & a \geq -\frac{1}{r}\left((z - \varepsilon_z)k^\alpha\right); \\ & k \geq 0. \end{aligned}$$

Assume the following parameter values:

1. Solve the problem for workers and plot welfare V^w and savings as univariate functions of initial wealth, permanent productivity y, and the variance of transitory shocks ε_y. Do optimal saving increase (or decrease) with initial wealth? Do optimal saving increase (or decrease) with permament productivity y? Do optimal savings increase (or decrease) with the variance of the transitory shocks ε_y? Discuss. What if $r = 0.01$?

2. Solve the problem for entrepeneurs and plot welfare V^e, savings a, and capital k as univariate functions of initial wealth, permanent productivity z, and the variance of transitory shocks ε_z. Do optimal a and k increase (or decrease) with initial wealth? Do optimal a and k increase (or decrease) with permament productivity z? Do optimal a and k increase (or decrease) with the variance of the transitory shocks ε_z? Discuss. What if $r = 0.01$?

3. Find the optimal threshold of initial wealth ω^* that separates the occupational choice.

   - How does permanent labor productivity y affect this threshold? To answer this plot the optimal ω^* (vertical axis) against different values of permament productivity y (horizontal axis).

- How does permanent entrepreneurial productivity z affect this threshold? To answer this plot the optimal ω^* (vertical axis) against different values of permament productivity z (horizontal axis).
- Is the ratio of productivities enough to pin down the optimal threshold ω^*?
- How does labor income risk affect this threshold? To answer this plot the optimal threshold ω^* (vertical axis) against different values of labor income risk ε_y (horizontal axis).
- How does entrepreneurial risk affect this threshold? To answer this plot the optimal ω^* (vertical axis) against different values of labor income risk ε_z (horizontal axis).

Question 2. Intrahousehold allocation of resources and welfare

Consider the following intrahousehold model model where wife's consumption is c_f and wife's labor supply is l_f. Analogously for the husband we have c_m and l_m. There is only one factor input, labor l that is elastically provided. The household agrees to solve:

$$\max_{c_m, c_f, l_m, l_f} log(c_m) + \lambda_m log(1 - l_m) + \delta_m[log(c_f) + \lambda_f log(1 - l_f)]$$

subject to:

$$c_m + c_f = y_m + y_f$$
$$y_i = z_i l_i^\alpha \text{ for } i \in \{m, f\}$$

where $\alpha < 1$. That is, each member of the household operates a decreasing returns to scale technology. Each household member has potentially different productivities z_i.

Assume the following parameter values:

Parameter	Value
δ	1.00
α	1/3
$z_f = z_m$	1.0885

- Report the value of consumption and hours worked separately for husband and wife.
- Report welfare $u(c_i) + \lambda log(1 - l_i)$ separately for husband and wife.
- Report the marginal products of labor separately for husband and wife. Do they equate? What does that imply?
- Compute model output, and compare it to the output generated from a model where there is no disutility of labor. Discuss your results.
- What happens to all your previous results if we decrease δ_m to 0.5?
- What happens to all your previous results if $z_f = 2z_m$?

Question 3. Computing Transitions in a Representative Agent Economy

Consider the following closed optimal growth economy populated by a large number of identical infinitely lived households that maximize:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}, \tag{2}$$

over consumption and leisure $u(c_t) = \ln c_t$, subject to:

$$c_t + i_t = y_t \tag{3}$$

$$y_t = k_t^{1-\theta} \ (z \ h_t)^{\theta} \tag{4}$$

$$i_t = k_{t+1} - (1-\delta) \ k_t \tag{5}$$

Set labor share to θ=.67. Also, to start with, set h_t=.31 for all t. Population does not grow.

1. Compute the steady-state. Choose z to match an annual capital-output ratio of 4, and an investment-output ratio of .25.
2. Double permanently the productiviy parameter z and solve for the new steady state.
3. Compute the transition from the first to the second steady state and report the time-path for savings, consumption, labor and output.
4. **Unexpected shocks.** Let the agents believe productivity z_t doubles once and for all periods. However, after 10 periods, surprise the economy by cutting the productivity z_t back to its original value. Compute the transition for savings, consumption, labor and output.
5. **Bonus Question: Can taxes explain differences in the speed of transition to steady-state?**
   - Add a permanent consumption tax. Recompute the new steady state, and the transitions.
   - Add a permanent capital tax. Recompute the new steady state, and the transitions.
6. **Bonus Question: Boldrin, Christiano and Fisher (AER, 2001) and Christiano (Minn QR, 1989)**
   - What if preferences take the form of Boldrin, Christiano and Fisher (AER, 2001)? That is, abstracting from labor choice,

     $$u(c) = \ln(c_t - bc_{t-1}). \tag{6}$$

     Recompute the transition as posed in Question 1.
   - What if preferences take the form of Christiano (Minn QR, 1989)? That is, abstracting from growth,

     $$u(c) = \ln(c_t - \bar{c}) \tag{7}$$

     Recompute the transition as posed in Question 1. Plot the differences in the time path of savings.

- Now, allow for growth, i.e., $z_t = z_0(1+\lambda_z)^t$, and replicate Christiano's Chart 1-4 for Japan, and extend the exercise to as many countries as you can (e.g. China, Taiwan, Korea, South Africa and Zambia). Get historical data for the U.K. (as long time series as you can), and replicate those Charts.

7. **Bonus Question: Labor Choice** Allow for elastic labor supply. That is, let preferences be

$$u(c_t, 1-h_t) = \ln c_t - \kappa \frac{h_t^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \tag{8}$$

and recompute the transition as posed in Question 1.