

Quantitative Macroeconomics
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Homework 1 [Extension], due Tue Sep 19

Question 1. Function Approximation II

Consider the following CES function $f(k, h) = \left((1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$ where σ is the elasticity of substitution (ES) between capital and labor and α is a relative input share parameter. Set $\alpha = 0.5$, $\sigma = 0.25$, $k \in [0, 10]$ and $h \in [0, 10]$

- Show that σ is the ES.
- Compute labor share for an economy with that CES production function assuming factor inputs face competitive markets.
- Approximate $f(k, h)$ using a 2-dimensional Chebyshev regression algorithm. Fix the number of nodes to be 20 and try Cheby polynomials that go from degree 3 to 15. For each case, plot the exact function and the approximation (vertical axis) in the (k, h) space.
- Plot the exact isoquants associated with the percentiles 5, 10, 25, 50, 75, 90 and 95 of output. Use your approximation to plot the isoquants of the your approximation. Plot the associated errors per each of these isoquant.
- For each case, show the associated approximation errors (vertical axis) in the (k, h) space.
- Redo using $\sigma = 5.00$ and $\sigma = 1.00$