

**Quantitative Macroeconomics**  
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Homework 1, due Thursday Sep 14

**Question 1. Function Approximation I**

1. Approximate  $f(x) = x^{321}$  with a Taylor series around  $\bar{x} = 1$ . Compare your approximation over the domain  $(0,4)$ . Compare when you use up to 1, 2, 5 and 20 order approximations. Discuss your results.
2. Approximate the *ramp function*  $f(x) = \frac{x+|x|}{2}$  with a Taylor series around  $\bar{x} = 2$ . Compare your approximation over the domain  $(0,6)$ . Compare when you use up to 1, 2, 5 and 20 order approximations. Discuss your results.
3. Approximate these three functions:  $e^{\frac{1}{x}}$ , the *runge function*  $\frac{1}{1+25x^2}$ , and the *ramp function*  $f(x) = \frac{x+|x|}{2}$  for the domain  $x \in [-1, 1]$  with:
  - Evenly spaced interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.
  - Chebyshev interpolation nodes and a cubic polynomial. Redo with monomials of order 5 and 10. Plot the exact function and the three approximations in the same graph. Provide an additional plot that reports the errors as the distance between the exact function and the approximand.
  - Chebyshev interpolation nodes and Chebyshev polynomial of order 3, 5 and 10. How does it compare to the previous results? Report your approximation and errors.