Accuracy of the Approximated Solution

Quantitative Macroeconomics

Raül Santaeulàlia-Llopis

MOVE-UAB and Barcelona GSE

Fall 2018

Introduction

Suppose that we are using the piecewise linear interpolation. How can we determine when the solution is accurate enough that no more points in the grid are needed?

Or, if we are using the Chebyshev approximation method, how do we determine that increasing the order of the polynomial would not lead to any significant improvement in accuracy?

Euler Equation Error Analysis

The numerically approximated Euler equation is:

$$u_c(c_t) \simeq \beta(1+r)E_t\left[u_c(c_{t+1})\right]$$

Then, one can define the relative approximation error ε_t as the value such that the equation holds exactly at t:

$$u_c(c_t(1-\varepsilon_t)) = \beta(1+r)E_t\left[u_c(c_{t+1})\right]$$

which we can solve for ε_t.

For example, if $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ is CRRA, then

$$\varepsilon_t = 1 - \frac{\left(\beta(1+r)E_t\left[u_c(c_{t+1})\right]\right)^{-\frac{1}{\sigma}}}{c_t}$$

A value of $\varepsilon_t = 0.01$ means that the agent is maing a mistake equivalent to \$1 for every \$100 consumed when choosing consumption and saving in period t. See Aruoba, Fernandez-Villaverde and Rubio-Ramirez (2006) for a comparison of various global solution methods of the growth model based on Euler Equation error analysis.

Note that when we approximate c_t with continuous methods (i.e., over the entire state space) we can plot ε_t over the entire state space and check in which areas we are doing better (or worse).

den Haan-Marcet Test

den Hann and Marcet (1994) provide a simple based on the Hansen J test of overidentifying restrictions. We apply the test to our Euler equation

$$u_c(c_t) = \beta(1+r)E_t\left[u_c(c_{t+1})\right]$$

which implies the residual:

$$\varepsilon_{t+1} = u_c(c_t) - \beta(1+r)E_t\left[u_c(c_{t+1})\right]$$

should not be correlated with any t- variable (or earlier than t), because the expectation at time t is conditional on everything observable up to then.

Therefore, we should have, for every t,

$$E_t \left[\varepsilon_{t+1} \otimes h(z_t) \right] = 0 \tag{1}$$

where $\otimes$ is the element-by-element product and the term $h(z_t)$ is a $r \times 1$ vector of functions of z_t that includes al lthe varibles in the inforation set of the agent at time t like $\{y_j, c_j, a_j\}_{j=0}^{t}$.

Clearly, this is true only for the *exact* solution, while a badly approximated solution will not satisfy this property.

One can obtain an estimate of the LHS of (1) through a simulation of lenght S of the model and the construction of the average sample:

$$X_S = \frac{\sum_{s=1}^{S} \varepsilon_{t+1}^s \otimes h(z_t^s)}{S}$$

where $\{\varepsilon_{t+1}^s, z_t^s\}_{s=1}^S$

It can be shown that, under mild conditions, $\sqrt{S}X_S \to N(0, V)$, and one can construct the appropriate quadratic from for the test statistics

$$SX_S' \widehat{V}_S^{-1} X_S \to \chi_r^2$$

where $\widehat{V}_S^{-1}$ is the inverse of some consisent estimate of V.

Some remarks are in order.

1. First, the test does not require any knowledge of the true solution, which is a big advantage.
2. Second, given a certain level of approximation in the solution, we can always find a number S large enough so that the approximation fails the accuracy test. This is not a problem when comparing solution methods, since one can fix the same S for both, or one can look for the smallest S such that the method fails the test and compare these thresholds. However, when we want to judge the accuracy of our unique solution, how large should S be? Its not clear: Den Haan and Marcet pick S to be 20 times larger than the typical sample period available.