Progressivity and Development*

Leandro De Magalhães       Enric Martorell
U. of Bristol              U. of Edinburgh

Raül Sántaeulàlia-Llopis
MOVE-UAB and Barcelona GSE

February 15, 2019

Abstract

Our contribution is threefold. First, using micro data from 21 countries including the poorest and the richest in the world—our aim is to reach 72 countries for which data are available—we document (i) a negative relationship between the ability to insure consumption against income shocks and economic development and (ii) a negative relationship between the degree of tax progressivity and the stage of development. Importantly, the computation of tax progressivity includes formal (public) and informal (private) net transfers across households, which is particularly relevant for poor countries. Second, to capture these facts we propose a macroeconomy with idiosyncratic income shocks in which agents accumulate physical and human capital (through learning-by-doing) and face a progressive income tax function that depends on the stage of development. Our goal is to assess whether tax progressivity can go a long way in explaining the larger ability to insure consumption in poor countries than in rich countries. Third, we use this economy to assess the role of tax progressivity in explaining income per capita and welfare differences across countries. Our preliminary results suggest an important role for progressivity in explaining income and welfare differences across countries.

1 Introduction

Our contribution is threefold. First, using micro data from a wide range (aiming at 72) countries including the poorest and the richest in the world we document:

(i) A negative relationship between the ability to insure consumption against income shocks and economic development and

*We thank the Social Sciences Grant from Fundación Ramón Areces for financial support.
(ii) A negative relationship between the degree of tax progressivity and the stage of development. Importantly, the computation of tax progressivity includes formal and informal (family) net transfers across households, which is particularly relevant for poor countries.

Second, to capture these facts we propose a macroeconomy with idiosyncratic income shocks in which agents accumulate physical and human capital (through learning-by-doing) and face a progressive income tax function that depends on the stage of development. We solve our model (a sequence of steady states) from poor to rich, that is, across different degrees of tax progressivity. The comparison across steady states resembles that in Conesa et al. (2009). Our calibration consists of country-specific elements such as aggregate productivity, human capital productivity and the degree of tax progressivity which depends on the stage of development. We find that our framework based on cross-country heterogeneity in tax progressivity is able to largely explain the higher ability to insure consumption in poor countries compared with rich countries.

Third, we use this economy to assess the role of tax progressivity in explaining income per capita and welfare differences across countries. We quantitatively assess the implications of informal and formal tax progressivity on income per capita differences by imposing the US progressivity on the rest of the world. Lower progressivity implies higher aggregate physical and human capital at the expense of social insurance.\footnote{Unlike Krueger and Ludwig (2016), we are not considering counteracting policies such as school subsidies as these are largely absent in poor countries.} Our preliminary results suggest an important role for progressivity in explaining income and welfare differences across countries which contributes to the literature on cross-country income per capita differences (Klenow and Rodríguez-Clare, 1997; Caselli, 2005; Lagakos et al., 2018).

2 Empirical Evidence

Our data comprises 21 countries with at least 2 years of representative household surveys. Austria, China, Croatia, Cyprus, Ethiopia, France, Germany, Hungary, India, Italy, Latvia, Malawi, Niger, Portugal, Slovakia, Slovenia, Spain, Tanzania, Uganda, United Kingdom, United States.

2.1 Transmission of Income Inequality to Consumption Inequality Across Time and Space

We are interested in the transmission of income inequality to consumption inequality across time and space. To investigate this, we focus on standard measures of this transmission. We use raw data on inequality as well as within-group measures of inequality that remove the between-group inequality generated by a set of deterministic observable variables. Specifically, the idea is that...
this within-group (or residual) variation captures changes in income and consumption that are not explained by a set of controls that include sex, age, education of the household head, household composition, (size and number of children), area of residency (rural/urban) and within-country regions. We remove this variation separately by year and country.

\[
\tilde{y}_{ict} = \alpha + f(x_{ict}; \Theta) + y_{ict}
\]

where \( \tilde{y} \) denotes the variable of interest in logs (e.g., consumption and income), \( x \) collects the controls, and the subscripts denote the household \( i \), country \( c \) and year \( t \). The measures of residual inequality are computed using the (logged) residuals \( y_{ict} \). We will use the variance of logs as our inequality measure, but we find similar insights if we use the Gini index or alternative measures.

We present our results in two ways. First we use the residual income and consumption as described. This is a standard measure of inequality. This measure allows us to compare inequality across countries in different levels of development. In our second measure we strip our measures from country-fixed effects. This gives us a better sense of how inequality changes as countries grow. This fixed effect estimate also help us deal with standard issues that arise when comparing inequality across countries. One of the main issues is differences in survey questions and methods (see Beagle et al. 2015), other issues may be differences in cultural or historical factors.

In Figure 1a we can see our first result. There is higher income inequality in poor Sub-Saharan countries (variances of log near or above 1) than in rich Western countries (mostly below 1). This is in keeping with the results in De Magalhães and Santaeulàlia-Llopis (2018) that compare Malawi and the US. China and India – on their transition into middle-income countries – have a similar level of income inequality as the most unequal Western countries. Moreover, both show a clear upward trend in their income inequality as they grow (compare the points IND04 and IND11 and CHN91 and CHN09 in Figure 1a). Thus suggesting a clear negative relationship between income inequality and level of development across countries and a positive relationship as a given country grows richer. The latter can be seen in Figure 1b, in which we present the variance of logs of income demeaned at the country level. Figure 1b gives a better sense of the relationship between economic growth and inequality as a country grows. Note that these two facts can be reconciled with a within country Kuznet curve (Fields (2002), Palma (2011)), which is not visible as we yet to include middle income countries from Latin America and South East Asia in our data set.

In Figure 1c we can see there is no clear relationship between levels of development and consumption inequality. The US seems to be a clear outlier as a rich Western country with high consumption inequality. Tanzania, Malawi and Ethiopia all have higher consumption inequality
Table 1: Transmission of Income to Consumption Inequality Across Time and Space

(a) Raw Inequality

<table>
<thead>
<tr>
<th></th>
<th>( Var(c_{it}) )</th>
<th>( Var(y_{it}) )</th>
<th>( \frac{Var(c_{it})}{Var(y_{it})} )</th>
<th>( Cov(c_{it}) )</th>
<th>( Corr(c_{it}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln GDPp.c.</td>
<td>0.025</td>
<td>0.027</td>
<td>0.001</td>
<td>0.053</td>
<td>0.055</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.005)</td>
<td>(0.152)</td>
<td>(0.004)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs. Country-Year</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>

(b) Within-Group (Residual) Inequality

<table>
<thead>
<tr>
<th></th>
<th>( Var(c_{it}) )</th>
<th>( Var(y_{it}) )</th>
<th>( \frac{Var(c_{it})}{Var(y_{it})} )</th>
<th>( Cov(c_{it}) )</th>
<th>( Corr(c_{it}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ln GDPp.c.</td>
<td>0.020</td>
<td>0.018</td>
<td>0.001</td>
<td>0.033</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.142)</td>
<td>(0.003)</td>
<td>(0.006)</td>
</tr>
<tr>
<td>Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( R^2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs. Country-Year</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
<td>75</td>
</tr>
</tbody>
</table>
than every Western country in our sample except for the US. China in the 1989 and 1990 as as well as Niger seem to have consumption inequality similar to the least unequal European countries. Once we look at consumption inequality after accounting for fixed effects, however, the relationship is clear. Figure 1d show a positive relationship between economic growth and consumption inequality.

We study three different statistics to measure the transmission of income to consumption inequality, i.e., three statistics that indicate whether markets are complete and household can insure their consumption. Complete markets tests can be derived not only for mean growth (à la Townsend (1994)) but also for higher order moments. In particular, we can use the variance (see Attanasio and Weber (1993)). Our first statistic is the ratio of the variance of (logged)
consumption to the variance of (logged) income. Specifically, the higher is the ratio of the variance of consumption to the variance of income, the lower is the ability to insure consumption. Indeed, this is what we find in Figure 2a1. The transmission of income inequality to consumption inequality is approximately 0.35 for the set of poorest countries, and this increases by almost three times to approximately 0.9 in the US. This increase is clear when comparing countries across different levels of development. It is not clear once we introduce country fixed effects and look at the relationship within countries in Figure 2a2, where there is no clear relationship. This lack of a clear relationship may be due to the small number of years for each country in our data set.

An alternative formulation of the complete markets tests is that the covariance of consumption and income should be zero. Hence, our second statistic is the covariance between consumption and income. In Figure 2b1 we can see no clear relationship between covariances across countries in different levels of development. Once we introduce fixed effects in Figure 2b2 there is a clear positive relationship. Insofar as the covariance is a measure of comovement, the result in Figure 2b is the more informative one as it tell us that there is considerably more comovement between income and consumption in richest countries than in the poorest countries.

Our third statistic is the correlation coefficient. The result is unambiguous. Rich countries have a higher correlation between income and consumption than poor countries (Figure 2c1) and the correlation between consumption and income increases as a country grows (Figure 2c2). The correlation of consumption and income is approximately 0.4 among the poorest countries and 50% larger in rich countries, 0.65.

All three statistics we use suggest there is a clear deterioration of insurance across stages of development. In other words, a negative relationship between social insurance and development. This complements the experimental evidence in Jakiela and Ozier (2016). This is true whether we compare countries at different levels of development, or whether we control for fixed effects and focus on within country changes in income and consumption. As a robustness of the within country relationship we measure the ratio and the correlation coefficient separately for three different countries for which we have yearly regional data: Malawi, China, and the US. Malawi is clearly among the poorest, China is a middle income country, and the US is one of the richest. The results are in Figure 3. The ratio of the variance of consumption inequality over income inequality increases with the level of development in all three countries, and so does the correlation between consumption and income.
Figure 2: The Transmission of Income Inequality to Consumption Inequality across Stages of Development

Notes: Computed by the authors
2.2 Tax Progressivity Across Time and Space

Our computation of tax progressivity includes formal and informal (family) net transfers across households, which is particularly relevant for poor countries. To study the degree of progressivity we use a class of tax policies traditional in public finance (Feldstein (1969)) defined by:

\[ T(y, Y) = y \left( 1 - \lambda(Y) y^{-\phi(Y)} \right), \]

where \( y \) is pre-tax income, \( T(y, Y) \) is the total tax (\( \tilde{y} = y - T(y, Y) \) is post-tax income). The parameters to be estimated are \( \lambda(Y) \geq 0 \), and \( \phi(Y) \geq 0 \). The parameter \( \lambda(Y) \) determines the net revenue and \( \phi(Y) \) the degree of progressivity. Importantly, notice that these parameters depend on the aggregate income per capita. That is, the degree of progressivity can change with development. This implies that disposable income is:

\[ y^d = (1 - \tau(y, Y)) y \]

where \( \tau(y, Y) = \frac{T(y, Y)}{y} \) is the average tax rate. This tax function has been recently used in quantitative macro with heterogeneous agents (Persson (1983), Benabou (2000), Benabou

\[ \text{Notes: Computed by the authors} \]

\[ \text{Two key restrictions are implicit in } T_y(y). \text{ First, it is either globally convex in income, if } \phi_y > 0, \text{ or globally concave, if } \phi_y < 0. \text{ As a result, marginal tax rates are monotonic in income. The same restriction applies to the average tax rate. Second, it does not allow for lump-sum transfers in cash, since } T_y(0) = 0. \]
(2002), and Heathcote et al. (2014)). In the United States, (Heathcote et al., 2012, 2017)) have estimated a degree of progressivity of 0.18.

Note also that we can write the Post-Tax/Pre-Tax Income Ratio as $\frac{\tilde{y}}{y} = 1 - \tau((y, Y)) = \lambda(Y) y^{-\phi(Y)}$. Hence, with data on post- and pre-tax income we can estimate $\lambda(Y)$ and $\phi(Y)$. In particular, taking logs we have the equation we estimate:

$$\ln \left(\frac{\tilde{y}}{y}\right) = \ln \lambda(Y) - \phi(Y) \ln y.$$

Our first estimates for Malawi using the LSMS-ISA 2016 incorporates food transfers (mainly maize), cash transfers, and other in-kind transfers. We find a degree of tax progressivity of 0.41. This is more than double the degree of progressivity in the United States as estimated in (Heathcote et al., 2012, 2017)). We are currently extending this analysis to the remaining countries. This result, however suggests a negative relationship between the degree of progressivity and the level of development. The poorest African countries have a more progressive redistribution system than the US. In Malawi this is mostly informal transfers between households instead of taxation.

In the estimation of tax progressivity we will isolate the part that is formal (public sector) and informal (private transfers) which speaks to the work of Attanasio and Ríos-Rull (2000). We formalize the decomposition of informal (family or peer pressure) and formal taxes, we define

$$1 - \tau(y, Y) = (1 - \tau_I(y^d, Y))(1 - \tau_F(y, Y)) \tag{2}$$

where $\tau_F$ denotes formal taxation and $\tau_I$ informal taxation. Disposable income $y^d$ is income after formal taxation, that is, $y^d = (1 - \tau_F(y))y$, where

$$\tau_F(y, Y) = \left(1 - \lambda_F(Y)y^{-\phi_F(Y)} \right), \tag{3}$$

and

$$\tau_I(y, Y) = \left(1 - \lambda_I(Y)y^{d-\phi_I(Y)} \right), \tag{4}$$

3 An Illustrative Two-Age OLG Model

At every period, $n$ individuals are born with an initial endowment $\omega_0t$ distributed according to an initial endowment distribution $\Psi(\omega_0t)$, and an initial level of human capital $s_0t$ which is the same
across all agents in the economy. Agents live for two periods which makes the total population alive in each period equal to \( L = 2n \). Agents also differ in labor income shocks in the second period that can take two values \( \varepsilon_{1t+1} \) and \(-\varepsilon_{1t+1}\), with .5 probability.

### 3.1 Household Problem

Each price-taker households solve this two-age model.

**First age \((a = 0)\).** For given \((k_{0t}, s_{0t})\), agents solve:

\[
\max_{\{c_{0t} \geq 0, 0 \leq h_{0t} \leq k_{1t}, s_{1t}\}} \left( \log(c_{0t} - \overline{c}) - \kappa \frac{K_{0t}}{1 + \frac{1}{\beta}} \right) + \sum_{\varepsilon_{1t+1}} \pi(\varepsilon_{1t+1}) \left( \log(c_{1t+1} - \overline{c}) - \kappa \frac{K_{1t+1}}{1 + \frac{1}{\beta}} \right)
\]

subject to a set of first-period constraints,

\[
c_{0t} + k_{1t} = w_{t}s_{0t} + \omega_{0t}
\]

\[
s_{1t} = zK_{0t} + (1 - \delta_{s})s_{0t}
\]

**Second age \((a = 1)\).** For given \((k_{1t}, s_{1t}, \varepsilon_{1t+1})\) agents solve

\[
\max_{\{c_{1t+1} \geq 0, 0 \leq h_{1t+1} \leq 1\}} \left( \log(c_{1t+1} - \overline{c}) - \kappa \frac{K_{1t+1}}{1 + \frac{1}{\beta}} \right)
\]

subject to a second-period constraint,

\[
c_{1t+1} = y_{d1} + (1 - \delta_{k})k_{1t}
\]

where \(y_{d1}\) is disposable income,

\[
y_{d1} = (1 - \tau(y_{1t+1}))(y_{1t+1})
\]

with pre-tax income,

\[
y_{1t+1} = w_{t+1}s_{1t}h_{1t+1}\varepsilon_{1t+1} + r_{1+1}k_{1t}
\]

The tax-subsidy scheme implies that above a given income threshold \( \overline{y} \) individuals pay a tax that depends on their income \( y_{1} \) and below that income threshold individuals receive a transfer.
Notice that there is labor income risk only in the second period. Initial $\omega_0$ and $s_0$ are given.

The tax code allows for tax progressivity as in HSV with:\textsuperscript{3}

$$\tau(y) = 1 - \lambda y^{-\phi}$$

(10)

where the parameter $\phi$ determines the degree of progressivity. This implies that the threshold of income above which individuals pay a tax, i.e., $\tau(y) \geq 0$, is $\overline{y} = \lambda^{\frac{1}{\phi}}$.

This means that we can write disposable income (8) as,

$$y^d_1 = \lambda y_1^{1-\phi}$$

(11)

**Firms.** A representative firm produces a consumption good with a CRS technology,

$$Y_t = B K_t^{1-\theta} N_t^\theta$$

with

$$K_t = \sum_a \sum_{i=1}^n k_{iat} = \sum_{i=1}^n k_{i1t} \text{ and } N_t = \sum_a \sum_{i=1}^n x_{iat}$$

where note that $k_{i1t}$ is chosen in the previous period, and $x_{iat} = s_{iat} h_{iat} \epsilon_{iat}$. This firm demands capital and labor in competitive markets $r_t = (1-\theta) \frac{Y_t}{K_t}$ and $w_t = \theta \frac{Y_t}{N_t}$.

**Aggregate Transfer Budget.** The economy satisfies an aggregate transfer budget constraint in the second period:

$$\sum_{i=1}^n 1_{y_i \geq \overline{y}} T(y_1) y_1 = \sum_{i=1}^n 1_{y_i < \overline{y}} T(y_1) y_1 + G$$

(12)

\textsuperscript{3}Recall that disposable income as

$$y^d = y - T(y)$$

where $y$ is pre-tax income and $T(y)$ is the total tax. We use Feldstein 1969 or HSV: $T(y) = y(1 - \lambda y^{-\phi})$ with $\lambda \geq 0$ and $\phi \geq 0$. Note than that we can write disposable income as

$$y^d = y - T(y) = (1 - \tau(y)) y = \lambda y^{1-\phi}$$

where we have used the fact that $\tau(y) = \frac{T(y)}{y} = 1 - \lambda y^{-\phi}$. 
with $G = G + W$. The amount $G$ can be interpreted as the provision of a public good (or rent-seeking resources and corruption). We start by setting $G$ its minimum, $G = 0$. In addition, part of the tax revenues are randomly allocated to the youngest individuals with $\omega_0 \sim N(0, \sigma_\omega^2)$ subject to the constraint that $\sum \omega_i = W$.

**Parameters.** We need to choose three preference parameters ($c, \kappa, \nu = 1$), two production parameters ($B, \theta = .64$), the labor income shock $\varepsilon$, two human capital parameters ($z, \alpha, s_0$), and the distribution of initial endowment $\Psi(\omega_0)$ and initial human capital $\Psi(h_0)$. We also need to choose the degree of tax progressivity $\phi$ and the size of the government budget through $\lambda$.

### 3.2 Stationary OLG Equilibrium

Given a tax system $\tau(y)$ (i.e., $\lambda$ and $\phi$), government expenditure ($G, W$), a joint initial distribution of initial wealth and schooling $\Phi(\omega_0, s_0)$, and a probability distribution $\pi(\varepsilon_1)$, a GE is a sextuplet $\{c_0, c_1, h_0, h_1, k_1, s_1\}$ of optimal choices, market wages ($w^*$) and interest rate ($r^*$) such that:

1. Given factor prices, households solve their maximization problem, that is, the sextuplet $\{c_0, c_1, h_0, h_1, k_1, s_1\}$ is the solution to the lifecycle problem (5)-(18).

2. Firms solve their optimization problem equating factor prices to marginal productivities.

3. Markets clear,

$$K^* = \sum_{i=1}^n k_1^*, \quad N^* = \sum_{i=1}^n x_a^*,$$

where $x^* = sh^*\varepsilon$.

---

4To see how $\lambda$ determines the size of public expenditure we use the budget constraint (12),

$$\sum_{i=0}^n 1_{y_i \geq y^}\tau(y)i = \sum_{i=0}^n 1_{y_i \geq y^}y - \lambda \sum_{i=0}^n 1_{y_i \geq y^}y^{1-\phi} = G.$$

Therefore, for a given distribution of income $\Phi(y)$, the higher is $\lambda$, the lower is aggregate amount of taxes collected, $\sum_{i=0}^n 1_{y_i \geq y^}\tau(y)i$, and hence the lower is public expenditure, $G$. First, an increase in $\lambda$ increases the income threshold ($y^ = \lambda^\phi$) above which population gets taxed which reduces the aggregate tax revenue. Second, because the aggregate amount of tax revenue is reduced, so is the aggregate amount of transfers. In particular, an increase in $\lambda$ increases the number of individuals that get transfers while at the same time reducing public expenditure $G$. Nevertheless, the distribution of income, $\Phi(y)$, potentially changes in equilibrium in response to $\lambda$ and this makes the effects of $\lambda$ on the size of the aggregate tax revenue (and aggregate transfers) ambiguous.
4. Government budget balances:

\[ \sum_{i=1}^{n} 1_{y_i^* \geq \tau} (y_i^*) y_i^* = \sum_{i=1}^{n} 1_{y_i^* < \tau} (y_i^*) y_i^* + G \]

where \( G = G + V \) with \( \sum_{i=1}^{n} \omega_0 = V \).

3.3 Solution Algorithm

We solve the problem with the following algorithm:

STEP 1. Guess the stationary interest rates \((r_m^*)\) (hence, \((w_m^*)\)).

STEP 2. Given factor prices, solve the household problem (where \(m\) stands for the iteration number). (See Appendix A).

STEP 3. Compute the excess of demand of aggregate capital and labor per period,

\[ K^* - \sum_{i=1}^{n} k_i^* = 0, \quad N^* - \sum_{a} \sum_{i=1}^{n} x_a^* = 0, \]

STEP 4. Check for the aggregate transfer budget balance,

\[ \sum_{i=1}^{n} 1_{y_i^* \geq \tau} (y_i^*) y_i^* = \sum_{i=1}^{n} 1_{y_i^* < \tau} (y_i^*) y_i^* + G \]

Notice that for the budget balance to clear we need to choose the appropriate \(\lambda\) (that is we need to also iterate over \(\lambda\) together with the \(r_m^*\) loop or outside)

STEP 5. If factor markets clear and government budget balances, then STOP. Otherwise guess a new interest rate and transfers.

3.4 Illustrative Numerical Results

We now illustrate the implications of tax progressivity in aggregate variables and distributions. To do so we choose some model parameters, \(A = 5.0, \theta = 0.66, z = 1.0, \alpha = 0.33, \delta_s = 0.001, \kappa = 5.0, \nu = 1.0, \bar{c} = 0.5\). We also assume that all individuals are born with the same initial human capital \(s_0 = 1.0\). In this manner, individuals differ in the amount of initial wealth they are born with \(k_0\) which is drawn from a log normal distribution with mean 0.0 and variance 0.1.
We now show the effects of changing tax progressivity from $\phi = 0.1$ to $\phi = 0.3$. This implies changing $\lambda$ to satisfy tax revenue neutrality across scenarios. In particular, the aggregate transfer scheme implies that 5% of all tax revenue is devoted is transferred as initial wealth to the youngest individuals with an evenly distributed lump sum.

The effects of tax progressivity on development are in Figure 4. Focusing on the effects from moving progressivity from 0.1 to 0.3, we find clear effects. The higher the progressivity lowers income per capita by 17.3% (see panel (a)). This is due to the drop in investment and aggregate capital (32.5%) and a smaller drop in efficient labor supply (-0.8%). Because there are no taxes for young individuals, a higher progressivity makes households work more in the first period and accumulate human capital which explains the rise in human capital (though small, 0.6%). The overall effects on hours is a decrease of 1.5% in response to increases in progressivity. This decline in aggregate hours is driven by the old adults that reduce hours supplied in response to increases in progressivity. The decline in consumption per capita is similar to that of output. Wages follow labor productivity that goes down with progressivity due to the larger decline of output than hours. The opposite occurs to the interest rate which increases due to the larger decline of capital than output.
The effects of progressivity show up in the average tax rate (ATR) across the income distribution, Figure 5. Higher progressivity from 0.1 to 0.3 increases the income subsidies received by the bottom 1% by 349.9% (from an ATR of -0.11% to -52.6%). On the other side of the income distribution the top 1% income earners see their ATR increase by 74.7% (from 0.08% to 14.2%). The increase in progressivity in turn implies that the fraction of tax payers goes down. Looking at the implications of increases in progressivity for the behavior of household \( y, x, k, h, s \) and \( c \) is also important.

Inequality in income and consumption is reduced with progressivity, see Figure 6. Increasing \( \phi \) from 0.1 to 0.3, the variance of logged income goes down by 6.5% and that of consumption by 20%. This implies a reduction in the inequality ratio between consumption and income of 14.4%, which is our first evidence of consumption insurance improvement due to increases in progressivity. This is directly related to the decrease in the variance of disposable income with respect to pre-tax income by 37.1% (from 0.81 with \( \phi = 0.1 \) to 0.51 with \( \phi = 0.3 \)). Part of the reduction in consumption inequality is related to the increase in the inequality of the labor supply that is also used as insurance mechanism.

An alternative measure of consumption insurance is the comovement of consumption and
Figure 6: Progressivity Effects on Inequality

(a) $\text{var}(y)$  
(b) $\text{var}(y^d)$  
(c) $\text{var}(c)$  
(d) $\text{var}(h)$  
(e) $\text{var}(s)$  
(f) $\text{var}(k)$

Notes: Computed by the authors

Figure 7: Progressivity Effects on Consumption Insurance

(a) $\text{cov}(\ln c, \ln y)$  
(b) $\text{cov}(\ln c, \ln \varepsilon)$  
(c) $\text{cov}(\ln wse, \ln h)$  
(d) $\text{cov}(\ln \varepsilon, \ln h)$

Notes: Computed by the authors
income. Clearly, higher progressivity implies lower covariance between consumption and income which is reduced by 25.4% (from 0.460 with $\phi = 0.1$ and 0.343 with $\phi = 0.3$), see panel (a) in Figure 7. A perhaps more direct measure of consumption insurance is the covariance between the income shock $\varepsilon$ and consumption. Again, the results are clear. Higher progressivity implies lower covariance between consumption and income shocks which is reduced by 21.1% (from 0.057 with $\phi = 0.1$ and 0.045 with $\phi = 0.3$), see panel (a) in Figure 7. Analogously, the higher is the progressivity the lower is the comovement between wages (or wage income shocks) and hours.

4 The Model

This is an OLG economy with $J$ generations. That is, at every period there is a continuum of ex-ante identical households being born that lives for $J$ periods. Let us cast the problem of these households recursively and then explain it. At any given age $j \in (0, J)$, agents with physical capital $k \in K$, human capital $s \in S$, labor productivity shock $\varepsilon \in \mathcal{E}$ solve the following problem,

$$V_t(k, s, \varepsilon, j, \Phi) = \max_{\{c \geq \bar{c}, 0 \leq h \leq 1, k' \geq k, s' \geq 0\}} \left( \log(c - \bar{c}) - \kappa \frac{h^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right) + \sum_{j=0}^{J} \beta^j \sum_{\varepsilon'} \pi(\varepsilon'|\varepsilon)V_{t+1}(k', s', \varepsilon', j+1, \Phi')$$

subject to individual constraints,

$$c + i = (1 - \tau(y))(w(\Phi)sh\varepsilon + r(\Phi)k)$$
$$i = k' - (1 - \delta_k)k$$
$$s' = z h^{\alpha} + (1 - \delta_s)s$$

and to the aggregate law of motion,

$$\Phi' = H(\Phi)$$

where the joint distribution of individual states $\Phi(k, s, \varepsilon, j)$ is the aggregate state of the economy which evolves following a law of motion $H$ defined below.

Households derive utility from consumption $c$ and leisure $1 - h$. We assume a subsistence level in consumption $\bar{c}$ which will be helpful later on to pin down the negative relationship between labor supply and development. Labor is supplied elastically with an elasticity with respect to effective wages determined through $\nu$. The degree of disutility of labor, relative to the joy of consumption, is guarded by $\kappa$. The future is discounted with a factor $\beta$.

The flow of household resources consists of labor income and capital income, which are taxed
at an endogenous rate $\tau(y)$ defined as,

$$\tau(y) = 1 - \lambda y^{-\phi}$$  \hspace{1cm} (18)

where the parameter $\phi$ determines the degree of progressivity. This implies that the threshold of income above which individuals pay a tax, i.e., $\tau(y) \geq 0$, is $\bar{y} = \lambda^{\frac{1}{\phi}}$.

Households differ in labor income through three different components: the level of human capital, labor supply, and a labor productivity shock. Each individual faces the same stochastic labor productivity process $\varepsilon \in \{\varepsilon_1, ..., \varepsilon_N\}$ that follows a stationary Markov process with conditional transition probabilities denoted by $\pi(\varepsilon' | \varepsilon)$.

Humcan capital is accumulated through a learning-by-doing that depends on the amount of labor supplied. The ability to accumulate human capital is defined by the parameter $z$ and its curvature by $\alpha$. Human capital depreciates at some rate $\delta_s$. There is investment in physical capital which will be rent out to firms in exchange of a common capital return. Physical capital is subject to a depreciation $\delta_k$.

In the beginning and end of life the problem is slightly different due to changes in the budget constraint. At age $j = 0$ agents are born without capital and their resources consist of after-tax labor earnings plus a lump sum transfer. That is, at age $j = 0$ the budget constraint is $c + k' = (1 - \tau_Y(y))w(\Phi)sh\varepsilon + \omega$. In the last period of life $k' = 0$ and households consume the entire income and wealth. That is, at age $J$, the budget constraint is $c = (1 - \tau(y))(w(\Phi)sh\varepsilon + r(\Phi)k) + (1 - \delta_k)k$.

There are four individual states: $\{k, s, \varepsilon, j\} \in K \times S \times E \times J$. The set $K = [k, +\infty)$ contains the possible asset holdings, $S = [0, +\infty)$ is the possible values of human capital, $E$ contains the possible realizations of the labor productivity shock, $J = \{0, J\}$ contains the ages where the first and last possible ages are 25 and 90, that is, $J = 65$. Define by $\mathcal{M}$ the set of all probability measures on the measurable space $\mathcal{M} = (Z, \mathcal{B}(Z))$ where $Z = K \times S \times E \times J$ and $\mathcal{B}(Z) = \mathcal{B}(K) \times \mathcal{B}(S) \times \mathcal{P}(E) \times \mathcal{P}(J)$. This is relevant because our measures $\Phi$ are required to be elements of $\mathcal{M}$.

The aggregate law of motion $H : \mathcal{M} \rightarrow \mathcal{M}$ maps distributions onto distributions. It basically summarizes how agents move within the distribution of physical assets, $k$, human capital, $s$,
income shocks, \( \varepsilon \), and age, \( j \), from one period to the next. Then, the evolution of the physical asset-human capital-productivity-age distribution is,

\[
\Phi'(K, S, E, J) = H(\Phi)(K, S, E, J) = \int_{k,s,\varepsilon,j} Q((k, s, \varepsilon, j))(K, S, E, J))d\Phi
\]

The fraction of people with assets in \( K \), human capital in \( S \), productivity shock in \( E \), and age \( J \), as measured by \( \Phi \), that transit to \(( K, S, E, J)\) as measured by \( Q \).

The evolution of the aggregate state is important because it provides a forecast of the evolution of the future rate of return on aggregate capital which is identical across households. Capital and labor demand are determined competitively by a representative firm that maximizes profits producing consumption goods using a constant returns to scale technology,

\[
Y = BK^{1-\theta}N^{\theta}t
\]

The competitive capital and labor market factor prices are \( r(\Phi) = (1 - \theta) \frac{Y}{K} \) and \( w(\Phi) = \theta \frac{Y}{N} \), respectively.

The economy satisfies an aggregate transfer budget constraint in every period:

\[
\int_{k,s,\varepsilon,j} 1_{y \geq \Phi}(y)yd\Phi = \int_{k,s,\varepsilon,j} 1_{y<\Phi}(y)yd\Phi + G
\]

with \( G = G + \mathcal{W} \). The amount \( G \) can be interpreted as the provision of a public good (or rent-seeking resources and corruption). We start by setting \( G \) its minimum, \( G = 0 \). In addition, part of the tax revenues are randomly allocated to the youngest individuals with \( \omega \sim N(0, \sigma^2_{\omega}) \) subject to the constraint that the youngest individual wealth is:

\[
\int_{k=0, s, \varepsilon, j=1} \omega d\Phi(k = 0, s, \varepsilon, j = 1) = \mathcal{W}.
\]

### 4.1 Stationary Recursive OLG Competitive Equilibrium

**Definition.** A stationary recursive OLG competitive equilibrium is a value function \( V : Z \to R \), policy functions for the household \( c : Z \to R \), \( h : Z \to R \), \( k' : Z \to R \) and \( s' : Z \to R \), policies for the firm \( K, L \), prices \( r, w \) and a measure \( \Phi \in \mathcal{M} \) such that,

1. That is exactly what a transition function tells us. Define the transition function \( Q : Z \times \mathcal{B}(Z) \to [0, 1] \) by

\[
Q((k, s, \varepsilon, j))(K, S, E, J)) = \left\{ \begin{array}{ll}
\pi(\varepsilon' | \varepsilon) & \text{if } g_{k}(k, s, \varepsilon, j; \Phi) \in K, g_{s}(k, s, \varepsilon, j; \Phi) \in S \text{ and } \varepsilon' \in E \\
0 & \text{else}
\end{array} \right.
\]

for all \((k, s, \varepsilon, j) \in Z \) and \((K, S, E, J) \in \mathcal{B}(Z) \). That is, \( Q((k, s, \varepsilon, j))(K, S, E, J)) \) is the probability that an agent with current physical assets \( k \), current huma capital \( s \), and current shock \( \varepsilon \) and current age \( j \) ends up with assets \( k' \) in \( K \) tomorrow, human capital \( s' \) in \( S \) tomorrow, income shocks \( \varepsilon \in E \) tomorrow, and age \( j' \) in \( J \) tomorrow.
1. $V, c, h, k'$ and $s'$ are measurable with respect to $B(Z)$, $V$ satisfies the household’s Bellman equation and $c, h, k', s'$ are the associated policy functions, given $r$ and $w$.

2. $K$ and $L$ satisfy, given $r$ and $w$,

$$r = F_K(K, L)$$
$$w = F_L(K, L)$$

3. Markets clear,

$$K = \int_{k,s,\varepsilon,j} k'(k, s, \varepsilon, j) d\Phi$$
$$N = \int_{k,s,\varepsilon,j} s'(k, s, \varepsilon, j) h(k, s, \varepsilon, j) \varepsilon d\Phi$$

and

$$\int_{k,s,\varepsilon,j} k'(k, s, \varepsilon, j) d\Phi + \int_{k,s,\varepsilon,j} k'(k, \varepsilon, j) d\Phi = F(K, N) + (1 - \delta)K$$

4. The economy satisfies the aggregate transfer budget:

$$\int_{k,s,\varepsilon,j} 1_{y \geq \overline{y}}(y) y d\Phi = \int_{k,s,\varepsilon,j} 1_{y < \overline{y}}(y) y d\Phi + \mathcal{G}$$

with $\mathcal{G} = G + \mathcal{W}$, and $\int_{k=0,s,\varepsilon,j=1} \omega d\Phi(k = 0, s, \varepsilon, j = 1) = \mathcal{W}$.

Note that value functions (decision rules) and prices are not any longer indexed by measures $\Phi$, all conditions have to be satisfied only for the equilibrium measure $\Phi$. The last requirement states that the measure $\Phi$ reproduces itself: starting with measure physical capital, human capital, productivities, and ages today generates the same measure tomorrow.

5. **Calibration**

The tax-subsidy parameters $\phi$ and $\lambda$ are estimated from the data (Section 2) to capture the degree of progressivity and the total tax level over GDP. The rest of parameters are the same across countries, except for those related to aggregate productivity $B$ and the parameters associated to the accumulation of human capital. The parameters associated with the human capital (learning-by-doing) parameter $z$, $\alpha$, and $\delta_s$ as well as the initial level of human capital $s_0$ are country-specific
Table 2: Calibration

<table>
<thead>
<tr>
<th></th>
<th>Poor</th>
<th>Middle</th>
<th>Rich</th>
<th>Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y/L</td>
<td>14.60</td>
<td>9.56</td>
<td>4.00</td>
<td>Output per capita</td>
<td>PWT</td>
</tr>
<tr>
<td>K/L</td>
<td>0.99</td>
<td>0.56</td>
<td>0.17</td>
<td>Capital per capita</td>
<td>PWT</td>
</tr>
<tr>
<td>S/L</td>
<td>1.27</td>
<td>1.28</td>
<td>1.34</td>
<td>Human Capital per capita</td>
<td>Barro-Lee</td>
</tr>
<tr>
<td>H/L</td>
<td>0.32</td>
<td>0.33</td>
<td>0.36</td>
<td>Hours per capita</td>
<td>Bick et al. 2018</td>
</tr>
<tr>
<td>C/L</td>
<td>14.55</td>
<td>9.53</td>
<td>3.99</td>
<td>Consumption per capita</td>
<td>PWT</td>
</tr>
<tr>
<td>Taxes/Y</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>Tax Revenue (% of GDP)</td>
<td>WDI</td>
</tr>
<tr>
<td>(\tau(y)) (Top 10%)</td>
<td>0.08</td>
<td>0.11</td>
<td>0.15</td>
<td>Average Tax Rate (10% richest)</td>
<td>Micro Data (Section 2)</td>
</tr>
<tr>
<td>(\tau(y)) (Bottom 10%)</td>
<td>-0.11</td>
<td>-0.03</td>
<td>-0.61</td>
<td>Average Tax Rate (10% poorest)</td>
<td>Micro Data (Section 2)</td>
</tr>
<tr>
<td>W/Y</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>Inheritance (% of GDP)</td>
<td></td>
</tr>
<tr>
<td>r</td>
<td>3.99</td>
<td>4.48</td>
<td>6.13</td>
<td>Rate of Return</td>
<td>Monge-Naranjo et al. 2019</td>
</tr>
<tr>
<td>w</td>
<td>6.06</td>
<td>4.11</td>
<td>1.74</td>
<td>Wages</td>
<td>Koh et al. 2019</td>
</tr>
</tbody>
</table>

Country-specific Parameters:
- \(B\): 10.0, 8.0, 5.0 Productivity
- \(z\): 1.0, 1.0, 1.0 Learning Productivity
- \(\alpha\): 0.33, 0.33, 0.33 Learning Curvature
- \(\delta_0\): 0.0, 0.0, 0.0 Depreciation of Human Capital
- \(s_0\): 1.0, 1.0, 1.0 Initial Human Capital
- \(\phi\): 0.1, 0.2, 0.35 Tax Progressivity
- \(\lambda\): 0.1, 0.2, 0.35 Budget Balancing

Common Parameters:
- \(\rho\): 0.00, 0.00, 0.00 Persistence of income shocks
- \(\text{var}(u)\): 0.20, 0.20, 0.20 Variance of income shocks
- \(\kappa\): 5.0, 5.0, 5.0 Leisure Preference Weight
- \(\tau\): 0.5, 0.5, 0.5 Subsistence Consumption Function
- \(\nu\): 1.00, 1.00, 1.00 Elasticity of Labor Supply
- \(\theta\): 0.66, 0.66, 0.66 Labor Share

and targeted to match the difference in the wage profiles across countries that documented in Lagakos et al. (2018). We use simulated methods of moments strategy to get this. Finally, for a given human capital quartuplet \((z, \alpha, \delta_0, s_0)\) that explains wage growth across countries we obtain an implied amount of labor in efficiency units \(N_t\), and therefore the implied \(B\) that is needed to match aggregate income per capita differences.
6 Quantitative Experiment: The Effects of Progressivity

We compute through counterfactuals the effects of progressivity on income per capita differences across countries and on welfare differences across countries. We impose the US progressivity to all countries and see the implications for income per capita (and income per hour).

6.1 Income per capita differences across countries

Rich country defined by high $B$ (20 times larger than poor countries), high $z$ (4 times larger than poor countries), high $T \approx 30\%$ (versus 20\% in poor countries), and low progressivity $\phi = .15$ (versus $\phi = .3$ in poor countries). We conducta counterfactual of changing $\phi$ in poor countries to $\phi$ in rich countries. Our preliminary results are depicted in Figure 8. We find that moving poor countries to the tax progressivity of rich countries implies an increase in income per capita of approximately $5.1/4.2-1=25\%$, which explains roughly 8\% of the total income per capita differences between rich and poor countries. The gain in income per capita generated by reducing income tax progressivity comes at the cost of loosing insurance, see Figure 9. Moving poor countries to the US tax progressivity increases the covariance between income and consumption substantially explaining approximately $1- (0.475-0.455)/(0.475–.325)=85\%$ of the difference between in consumption insurance between rich and poor countries.
6.2 Welfare differences across countries

Figure 10 shows the welfare gains for a poor country of moving from its status quo to an alternative were we impose on poor countries the tax progressivity in the U.S. The implied increase in income (and consumption) per capita rises welfare, the associated decline in labor supply also increases income, while the loss of consumption insurance reduces welfare. We find that the first two effects dominate the loss in insurance. For all levels of initial wealth agents would prefer the alternative to the status quo being the individuals that gain the most the poorest in initial wealth equivalent to a 2.5% increase in consumption across periods and states of the world. For initially rich households these welfare gains are equivalent to an increase of 0.5% across periods and states of the world.

We need to decompose the effects of consumption and leisure, and the effects of growth versus insurance. (TO BE COMPLETED).

6.3 Tax Progressivity Versus Other Sources

- Compare the effects of imposing US progressivity on poor countries, versus imposing U.S. productivity $B$, or US human capital technology $(z, \alpha, \delta, s_0)$.

- Tax progressivity as a propagation mechanism. We complement U.S. tax progressivity with U.S. human capital technology (or productivity).
7 Decomposing Private and Public Transfers

Here we isolate the part that is formal (public sector) and informal (private transfers). We find that the role of private (public) transfers decreases (increases) with development which speaks to the work of Attanasio and Ríos-Rull (2000) regarding the crowding-out effect of private transfers due to public transfers. We formalize the decomposition of informal (private) taxes from family (or peers and neighbours) and formal (public) taxes from government. To do so we define

\[ 1 - \tau(y, Y) = (1 - \tau_I(y^d, Y))(1 - \tau_F(y, Y)) \]  
\[ (22) \]

where \( \tau_F \) denotes formal taxation and \( \tau_I \) informal taxation. Disposable income \( y^d \) is income after formal taxation, that is, \( y^d = (1 - \tau_F(y))y \), where

\[ \tau_F(y, Y) = \left(1 - \lambda_F(Y)y^{-\phi_F(Y)}\right), \]  
\[ (23) \]

and

\[ \tau_I(y, Y) = \left(1 - \lambda_I(Y)y^{d-\phi_I(Y)}\right), \]  
\[ (24) \]

Notes: Computed by the authors
Notes: Computed by the authors

8 Extension to 2-Good-2-Sector Economy

The composition of consumption and value added implies a drop in the amount of food consumption as the economy develops, see Panel (a) in Figure 11. This decomposition is relevant to understand the evolution of consumption insurance with development if the degree of insurance depends on the composition of the consumption basket, which we find it does. The rise in the covariance of nondurable consumption and income with development (Panel (d)) from 0.1
to 0.6 between rich and poor countries, is largely driven by the rise in the covariance of nonfood consumption (from 0.1 to 0.8). While that the rise in the covariance of food consumption and income is one fourth less from 0.2 to 0.4 between rich and poor countries.

We incorporate these elements into our model using the following structure for preferences,

$$u(c_a, c_m, h) = \log(c_a - \bar{c}_a) + \gamma \frac{c_m^{1-\sigma}}{1 - \sigma} - \kappa \frac{h^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}}$$

The idea is that the preferences penalize risk in $c_a$ more than risk in $c_m$.

We also modify the budget constraint,

$$p_a c_a + c_m + k' = (1 - \tau(y))y + (1 - \delta)k$$

where pre-tax income incorporates income from the production of both sectors,

$$y = \sum_{a, m} w_a h + r k$$

with both sectors following a CRS technology and equalitazion of factor prices across sectors $w = w_a = w_m$ and $r = r_a = r_m$.

We now reconduct our quantiative experiments in Section 6 using our 2-good-2-sector.

(TO BE COMPLETED)

9 Conclusion

Our preliminary results with a one-good-one-sector economy suggest that a higher progressivity of transfers helps explain the relatively higher insurance in poor economies. We show this progressivity has first order implications on the cross-country differences in income per capita explaining approximately 8% of these differences and larger differences in welfare. We are currently decomposing the effects generated from informal (private) transfers more prominent in poor countries and formal (public) transfers more prominent in rich countries. Also, motivated by the higher insurance in food consumption (relative to non-food consumption) we are also extending the economy to a 2-goods-2-sector economy to assess the relationship of insurance and structural change.
References


A Two-Period Model: Solution Algorithm for Households (Backwards)

This is a lifecycle model with uncertainty. We solve the problem backwards from last (second) period to the initial (first) period. In addition, to solve the problem we choose to plug $c_0, c_1$ and $s_1$ into the per-period objective functions. Our households take factor prices $w$ and $r$ as given.

Second Period. In the second period, for given $(k_1, s_1, \varepsilon_1)$, agents solve

$$\max_{\{0 \leq \varepsilon_1 \leq 1\}} \left( \log \left( y_1^d + (1 - \delta_k)k_1 - \varepsilon_1 \right) - \frac{h_1^{1+\frac{1}{\bar{p}}}}{1 + \frac{1}{\bar{p}}} \right)$$

(25)

where disposable income is,

$$y_1^d = \lambda y_1^{1-\phi},$$

with pre-tax income,

$$y_1 = w_1(zh_0^\alpha - (1 - \delta_s)s_0)h_1\varepsilon_1 + r_1k_1.$$

This implies the following FOC($h_1$) for the second period:

$$FOC(h_1) : \left( \begin{array}{l} \frac{1}{c_1 - \varepsilon} \left( \frac{\partial c_1}{\partial y_1} \right) (1 - \phi)\lambda y_1^{-\phi} w_1 s_1 \varepsilon_1 - \frac{\kappa h_1^{\frac{1}{1+\bar{p}}}}{1 + \frac{1}{\bar{p}}} \\
\text{\text{MU}(c_1)} \end{array} \right) = 0$$

(26)

with $c_1 = \lambda y_1^{1-\phi} + (1 - \delta_k)k_1$ and $y_1 = w_1(zh_0^\alpha - (1 - \delta_s)s_0)h_1\varepsilon_1 + r_1k_1$. Notice that we solve this FOC for each triplet and all triplets $(k_1, s_1, \varepsilon_1)$.

Remark. Notice that the choice of $h_1$ depends on the values of $k_1, s_1$ and $\varepsilon_1$. Clearly, at this point we do not know the optimal $(k_1, s_1)$ because these will be chosen in the previous period. For this reason, when solving for $h_1$ we do it for all feasible pairs $(k_1, s_1)$. In terms of timing we assume that the shock $\varepsilon_1$ is realized after the choices $k_1$ and $s_1$ are done, and before $h_1$ is chosen. This implies that we solve for $h_1$ in (26) for each and all triplets $(k_1, s_1, \varepsilon_1)$.

First Period. In the first period, for given $(k_0, s_0)$, agents solve

$$\max_{\{0 \leq \varepsilon_1 \leq 1\}} \left( \log \left( \frac{w_0 s_0 h_0 + r_0 k_0 - k_1}{c_0} - \varepsilon_1 \right) - \frac{h_1^{1+\frac{1}{\bar{p}}}}{1 + \frac{1}{\bar{p}}} \right)$$

29
\[ + \beta \sum_{\varepsilon_1} \pi(\varepsilon_1) \left( \log \left( \frac{y_1^d + (1 - \delta_k)k_1 - \varepsilon}{c_1 \geq 0} \right) - \kappa \frac{h_1^{1 + \frac{1}{c_1}}}{1 + \frac{1}{c_1}} \right) \]

where disposable income is,
\[ y_1^d = \lambda y_1^{1 - \phi}, \]

with pre-tax income,
\[ y_1 = w_1(z h_0^a - (1 - \delta_s)s_0)h_1 \varepsilon_1 + r_1 k_1. \]

This implies that households face these two first order conditions with two unkowns \( h_0 \) and \( k_1 \):

\[ \text{FOC}(h_0): \quad \frac{1}{c_0 - \bar{c}} \left. \frac{w_0 s_0}{MU(c_0)} \right|_{\varepsilon_1} + \frac{k h_0^{1/2}}{\mu(c_1)} + \beta \sum_{\varepsilon_1} \pi(\varepsilon_1) \left( \frac{1}{c_1 - \bar{c}} \right) \left. \frac{1}{MU(c_1)} \right|_{\varepsilon_1} \left( 1 - \phi \right) y_1^{-\phi} w_1 h_1 \varepsilon_1 + \alpha z h_0^{\alpha - 1} \right] = 0 \]

(27)

\[ \text{FOC}(k_1): \quad \frac{1}{c_0 - \bar{c}} \left. \frac{(-1)}{MU(c_0)} \right|_{\varepsilon_1} + \beta \sum_{\varepsilon_1} \pi(\varepsilon_1) \left. \frac{1}{c_1 - \bar{c}} \right|_{\varepsilon_1} \left( \frac{1}{MU(c_1)} \right) \left( 1 - \phi \right) y_1^{-\phi} w_1 h_1 \varepsilon_1 + \alpha z h_0^{\alpha - 1} \right] = 0 \]

(28)

with \( c_0 = w_0 s_0 h_0 + r_0 k_0 - k_1, c_1 = \lambda y_1^{1 - \phi} + (1 - \delta_k)k_1 \) and \( y_1 = w_1(z h_0^a - (1 - \delta_s)s_0)h_1 \varepsilon_1 + r_1 k_1. \)

\textbf{Remark.} Notice that the system (27)-(28) needs to be solved as many times as the number of initial conditions (i.e., pairs \((k_0, s_0)\)). Also, notice that to solve for the system (27)-(28) we make use of the optimal allocation, \( h_1 \), obtained earlier for the next period from (26) defined for each triplet \((k_1, s_1, \varepsilon_1)\).

\section*{B Data}

\section*{C Decision Rules}
<table>
<thead>
<tr>
<th>Countries</th>
<th>GDP p.c. (2010)</th>
<th>Year</th>
<th>Source</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albania</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Angola</td>
<td></td>
<td></td>
<td>IBEP</td>
<td></td>
</tr>
<tr>
<td>Armenia</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Benin</td>
<td></td>
<td></td>
<td>EMICOV</td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Bolivia</td>
<td></td>
<td></td>
<td>RIGA</td>
<td></td>
</tr>
<tr>
<td>Bosnia-Herzegovina</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Botswana</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Brazil</td>
<td></td>
<td></td>
<td>PNAD</td>
<td></td>
</tr>
<tr>
<td>Bulgaria</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Burkina Faso</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Cambodia</td>
<td></td>
<td></td>
<td>CSES</td>
<td></td>
</tr>
<tr>
<td>Chile</td>
<td></td>
<td></td>
<td>CASEN</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>1989-2009</td>
<td></td>
<td>CHNS</td>
<td></td>
</tr>
<tr>
<td>Colombia</td>
<td></td>
<td></td>
<td>GEIH</td>
<td></td>
</tr>
<tr>
<td>Cyprus</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Ecuador</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Egypt</td>
<td></td>
<td></td>
<td>LMPN</td>
<td></td>
</tr>
<tr>
<td>El Salvador</td>
<td></td>
<td></td>
<td>PHC</td>
<td></td>
</tr>
<tr>
<td>Estonia</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Ethiopia</td>
<td>2011, 2013, 2014</td>
<td></td>
<td>LSMS-ISA</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Ghana</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>2004, 2011</td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Indonesia</td>
<td></td>
<td></td>
<td>NLFS</td>
<td></td>
</tr>
<tr>
<td>Iraq</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Ivory Coast</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Jamaica</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Jordan</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Kenya</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Kyrgyzstan</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Laos</td>
<td></td>
<td></td>
<td>ECS</td>
<td></td>
</tr>
<tr>
<td>Latvia</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Luxembourg</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Mali</td>
<td></td>
<td></td>
<td>PHC</td>
<td></td>
</tr>
<tr>
<td>Malta</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Mauritius</td>
<td></td>
<td></td>
<td>CPHS</td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Mongolia</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Namibia</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Nicaragua</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Niger</td>
<td>2011, 2014</td>
<td></td>
<td>LSMS-ISA</td>
<td></td>
</tr>
<tr>
<td>Nigeria</td>
<td></td>
<td></td>
<td>LSMS-ISA</td>
<td></td>
</tr>
<tr>
<td>Pakistan</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Panama</td>
<td></td>
<td></td>
<td>LFS</td>
<td></td>
</tr>
<tr>
<td>Paraguay</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Peru</td>
<td></td>
<td></td>
<td>LHFS</td>
<td></td>
</tr>
<tr>
<td>Philippines</td>
<td></td>
<td></td>
<td>LF</td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td></td>
<td></td>
<td>HFS</td>
<td></td>
</tr>
<tr>
<td>Rwanda</td>
<td></td>
<td></td>
<td>EICVM</td>
<td></td>
</tr>
<tr>
<td>Serbia (and Montenegro)</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td></td>
<td></td>
<td>Census 2001</td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>2002-2014</td>
<td></td>
<td>EFF</td>
<td></td>
</tr>
<tr>
<td>Tajikistan</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td></td>
<td></td>
<td>Townsend Villages</td>
<td></td>
</tr>
<tr>
<td>Timor-Leste</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
<tr>
<td>Tunisia</td>
<td></td>
<td></td>
<td>ENPE</td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td></td>
<td></td>
<td>HLFS</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td></td>
<td></td>
<td>CEX, PSID</td>
<td></td>
</tr>
<tr>
<td>United States</td>
<td>1980-2012</td>
<td></td>
<td>PHC</td>
<td></td>
</tr>
<tr>
<td>Vietnam</td>
<td></td>
<td></td>
<td>LSMS</td>
<td></td>
</tr>
</tbody>
</table>
Figure 12: Progressivity Effects on Consumption

(a) Young’s consumption $c_0$
(b) Old’s consumption $c_1$
(c) Consumption per capita $\frac{C}{N}$

Notes:
Figure 13: Progressivity Effects on Hours

Notes:
Figure 14: Progressivity Effects on Income

(a) Young’s income $y_0$

(b) Old’s income $y_1$

(c) Income per capita $\frac{Y}{N}$

(a) Variance $\sigma^2_0$

(b) Variance $\sigma^2_1$

(c) Variance $\sigma^2_Y$

Notes:
Figure 15: Progressivity Effects on Human Capital

(a) Young’s human capital $s_0$

(b) Old’s human capital $s_1$

(c) Human Capital per capita $\frac{S}{N}$

(a) Variance $s_0$

(b) Variance $s_1$

(c) Variance $s$

Notes:
Figure 16: Progressivity Effects on Disposable Income

(a) Young's disposable income $y_0^d$

(b) Old's disposable income $y_1^d$

(c) Disposable income per capita $y^d$

(a) Variance $y_0^d$

(b) Variance $y_1^d$

(c) Variance $y^d$

Notes:
Figure 17: Progressivity Effects on Wages and Interest Rates

(a) Wage Rate $w$

(b) Interest Rate $r$

Notes:
Figure 18: Progressivity Effects on Wage Income

(a) Young’s wage income $w_{s_0} h_0$

(b) Old’s wage income $w_{s_1} h_1 \varepsilon$

(c) Wage income per capita $y^d$

Notes:
Figure 19: Progressivity Effects on Average Tax Rate

(a) Bottom 1% $\tau(y)$

(b) Bottom 10% $\tau(y)$

(c) Median $\tau(y)$

(d) Top 10% $\tau(y)$

(e) Top 1% $\tau(y)$

(c) Average $\tau(y)$

Notes:
Figure 20: Progressivity Effects on Covariance of Consumption and Income

(a) Young’s $cov(c_0, y_0)$
(b) Old’s $cov(c_1, y_1)$
(c) $cov(c, y)$

Notes:
Figure 21: Progressivity Effects on Covariance of Consumption and Transitory Income Shocks

(a) $\text{cov}(c_1, \varepsilon)$

(b) $\text{cov}(\ln c_1, \ln \varepsilon)$

Notes:
Figure 22: Progressivity Effects on Covariance of Wages and Hours

(a) Young’s $\text{cov}(w_{0}, h_{0})$

(b) Old’s $\text{cov}(w_{1}\varepsilon_{1}, h_{1})$

(c) $\text{cov}(w\varepsilon, h)$

Notes:
Figure 23: Progressivity Effects on Covariance of Hours and Transitory Income Shocks

(a) $\text{cov}(h_1, \varepsilon)$

(b) $\text{cov}(\ln h_1, \ln \varepsilon)$

Notes:
Figure 24: Productivity Effects on Consumption

(a) Young’s consumption $c_0$

(b) Old’s consumption $c_1$

(c) Consumption per capita $\frac{C}{N}$

(a) Variance $c_0$

(b) Variance $c_1$

(c) Variance $c$

Notes:
Figure 25: Productivity Effects on Hours

(a) Young’s hours $h_0$
(b) Old’s hours $h_1$
(c) Hours per capita $\frac{H}{N}$

(a) Variance $h_0$
(b) Variance $h_1$
(c) Variance $h$

Notes:
Figure 26: Productivity Effects on Income

(a) Young’s income $y_0$
(b) Old’s income $y_1$
(c) Income per capita $\frac{Y}{N}$

Notes:
Figure 27: Productivity Effects on Human Capital

(a) Young’s human capital $s_0$

(b) Old’s human capital $s_1$

(c) Human Capital per capita $\frac{S}{N}$

Notes:
Figure 28: Productivity Effects on Disposable Income

(a) Young’s disposable income $y_0^d$
(b) Old’s disposable income $y_1^d$
(c) Disposable income per capita $y^d$

(a) Variance $y_0^d$
(b) Variance $y_1^d$
(c) Variance $y^d$

Notes:
Figure 29: Productivity Effects on Wages and Interest Rates

Notes:
Figure 30: Productivity Effects on Wage Income

(a) Young’s wage income $w_s h_0$

(b) Old’s wage income $w_s h_1 \varepsilon$

(c) Wage income per capita $y^d$

(a) Variance $w_s h_0$

(b) Variance $w_s h_1$

(c) Variance $w_s h \varepsilon$

Notes:
Figure 31: Productivity Effects on Average Tax Rate

Notes:
Figure 32: Productivity Effects on Covariance of Consumption and Income

(a) Young’s $cov(c_0, y_0)$

(b) Old’s $cov(c_1, y_1)$

(c) $cov(c, y)$

(a) Young’s $cov(ln c_0, ln y_0)$

(b) Old’s $cov(ln c_1, ln y_1)$

(c) $cov(ln c, ln y)$

Notes:
Figure 33: Productivity Effects on Covariance of Consumption and Transitory Income Shocks

(a) $\text{cov}(c_1, \varepsilon)$

(b) $\text{cov}(\ln c_1, \ln \varepsilon)$

Notes:
Figure 34: Productivity Effects on Covariance of Wages and Hours

(a) Young’s $\text{cov}(w_0, h_0)$

(b) Old’s $\text{cov}(w_1 \varepsilon_1, h_1)$

(c) $\text{cov}(w \varepsilon, h)$

Notes:
Figure 35: Productivity Effects on Covariance of Hours and Transitory Income Shocks

Notes: