Policy Evaluation

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1. The Problem: Unexpected Policy Change and Transitional Dynamics

2. Definition of Equilibrium

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Introduction

- In our previous set of slides, the PILCH models (i.e., Ayagari-Bewley-Hugget economies) were stationary: The distribution of wealth (hence consumption) was invariant to time.
  - ▷ A stationary distribution of wealth implies aggregate output, investment, and consumption is also invariant to time.
  - ▷ That is, the model was silent about the how aggregate variables and distributions evolve over time (either with growth or over the business cycle).
- Here, we introduce one form of nonstationary economy:
  - ▷ The economy is not stationary along the transition to a stationary economy. This transition is generated by an unexpected shock to the environment of the economy (i.e., a policy change, demographic changes, an aggregate productivity shock, etc.)
  - ▷ There are other forms of nonstationary economies such as those derived from aggregate productivity shocks that hit the economy at every period. This is the Krusell and Smith (1998) economy. We will ignore these type of economies in these slides (check the Quantitative Macro course).

The Problem: Unexpected Policy Change and Transitional Dynamics

1. Suppose the economy is in a stationary equilibrium, given a government policy, preferences, endowments (labor earnings process) and technology.

2. Suppose there is an unexpected change (a zero probability event) in one of the exogenous elements in the model: government policy.

3. We want to study the transition path induced by the exogenous change, from the old stationary equilibrium to the new one.

- For instance, suppose an unexpected permanent introduction of a capital income tax at rate τ. The receipts are rebated lump-sum to households as government transfers, T.

- The initial policy is characterized by $\tau = T = 0$.

- Individuals are going to change the savings behavior and there will be a nontrivial transition path induced by the reform.

- Since the transition path is characterized by a sequence of prices, quantities and distributions we will cast the definition and solution of the model in sequential notation—the household problem still in recursive formulation.

- Let $Z = Y \times R_+$ be the set of all possible (y_t, a_t).

- Let $\mathcal{B}(R_+)$ be the Borel σ-algebra of R_+ and $\mathcal{P}(Y)$ the power set of Y.

- Let $\mathcal{B}(Z) = \mathcal{P}(Y) \times \mathcal{B}(R_+)$ and M be the set of all finite measures on the measurable space $(Z, \mathcal{B}(Z))$.

- The household problem is,

$$v_t(a, y) = \max_{c \geq 0, a' \geq 0} u(c) + \beta \sum_{y' \in Y} \pi(y'|y) v_{t+1}(a', y') \tag{1}$$

subject to

$$c + a' = w_t y + (1 + (1 - \tau_t) r_t) a + T_t \tag{2}$$

- The value functions are now functions of time because aggregate prices and polices change over time.

Definition (The competitive equilibrium)

Given the initial distribution Φ_0 and a fiscal legislation $\{\tau_t\}_{t=0}^{\infty}$, a competitive equilibrium is a sequence of individual household functions $\{v_t, c_t, a_{t+1} : Z \times M \rightarrow R\}_{t=0}^{\infty}$, sequence of production plans $\{N_t, K_t\}_{t=0}^{\infty}$, factor prices $\{w_t, r_t\}_{t=0}^{\infty}$, government transfers $\{T_t\}_{t=0}^{\infty}$ and a sequence of measures $\{\Phi\}_{t=1}^{\infty}$ such that, $\forall t$,

1. Given $\{w_t, r_t\}$ and $\{T_t, \tau_t\}$ the functions v_t solve Bellman's equation for period t and c_t, a_{t+1} are the associated policy functions.
2. Factor prices $\{w_t, r_t\}$ satisfy $w_t = F_L(K_t, L_t)$ and $r_t = F_K(K_t, L_t) - \delta$.
3. Balanced Government Budget: $T_t = \tau_t \; r_t \; K_t$.
4. Market Clearing

$$\begin{aligned} \int c_t(y_t, a_t) d\Phi_t + K_{t+1} &= F(K_t, L_t) + (1-\delta)K_t \\ L_t &= \int y_t d\Phi_t \\ K_{t+1} &= \int a_{t+1}(y_t, a_t) d\Phi_t \end{aligned}$$

5. Aggregate Law of Motion: $\Phi_{t+1} = \Gamma_t \Phi_t$.

- **A Stationary Equilibrium** is an equilibrium such that all elements of the equilibrium that are indexed by t are constant over time.

Computation of the Equilibrium Transition Path

- What we are after:

  1. At $t = 0$ we have a stationary equilibirum with τ_0 and associated equilibrium distribution Φ_0 (hence we have K_0, r_0, w_0) and associated value function v_0 and decision rules c_0, a_1.

  2. At $t = 1$ policy changes permanently to $\tau_t = \tau > 0$ for all $t \geq 1$.

  3. Denote the new stationary equilibrium associated with τ by Φ_∞ with associated value function v_∞ and decision rules c_∞, a_∞.

- We want to compute the entire transition path and compute the welfare consequences of such policy innovation.

- How can we compute the transition path?
  - ▷ Assume it takes T periods to move from the old stationary equilibrium to the new one.
  - ▷ T should be sufficiently large so that the new stationary is reached.
  - ▷ Using the fact that $v_T = v_\infty$, then for a given sequence of prices $\{r_t, w_t\}_{t=1}^{T}$ the household problem can be solved backwards. This is independent of whether people leave forever or not!

Algorithm (Computing the transitional dynamics)

1. *Fix T.*
2. *Compute stationary equilibrium at $t = 0$.*
3. *Compute stationary equilibrium at $t = \infty$ assuming that stationarity is reached at $t = T$.*
4. *Guess a sequence of prices and transfers choosing $\{\hat{K}_t\}_t^{T-1}$ (note that $\hat{K}_1 = K_0$ and $L_t = L_0 = \bar{L}$ is fixed):*
$$\hat{w}_t = F_L(\hat{K}_t, \bar{L}),\ \hat{r}_t = F_K(\hat{K}_t, \bar{L}),\ \text{and}\ \hat{T}_t = \tau_t \hat{r}_t \hat{K}_t.$$
5. *Since we know $v_T(a, y)$ and $\{\hat{r}_t, \hat{w}_t, \hat{T}_t\}_{t=1}^{T-1}$ we can solve for $\{\hat{v}_t, \hat{c}_t, \hat{a}_{t+1}\}_{t=1}^{T-1}$ backwards.*
6. *With the sequence for $\hat{a}_{t+1}$ we can define the transition laws $\{\hat{\Gamma}_t\}_{t=1}^{T-1}$. Since we know that $\Phi_1 = \Phi_0$ from the initial stationary equilibrium, we can iterate the distributions forward*
$$\hat{\Phi}_{t+1} = \hat{\Gamma}_t \hat{\Phi}_t$$
7. *With $\{\hat{\Phi}_t\}_{t=1}^{T}$ we can compute*
$$\hat{A}_t = \int a \, d\hat{\Phi}_t$$
8. *Check whether*
$$\max_{1 \le t < T} |\hat{A}_t - \hat{K}_t| < \varepsilon$$
If yes, go to 9. If not, adjust your guesses for $\{\hat{K}_t\}_{t=1}^{T-1}$ in step 4.
9. *Check whether $|\hat{A}_T - \hat{K}_T| < \varepsilon$. If yes, we are done. If not, go back to step 1 and adjust T.*

- It turns out that with the sequence of value functions v_t we can make statements about welfare. What are the welfare consequences of a tax reform?

Measuring Welfare Consequences of Policy

To measure the welfare consequences of unexpected policy change we need to take into account the entire transtion path.

- The welfare consequences are a result of interpreting value functions:
  - ▷ Function $v_0(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 0 in the initial stationary equilibrium—i.e., for a person that thinks he will live in the stationary equilibrium with $\tau = 0$ forever.
  - ▷ Function $v_1(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 1 that has just been informed that there is a permanent tax change—i.e., this $v_1(a, y)$ takes into account all the transition dynamics through which the agent is going to live.
  - ▷ Function $v_T(a, y) = v_\infty(a, y)$ is the expected lifetime utility of an agent with assets a and productivity shock y at time 0 in the final stationary equilibrium—i.e., this agent does not live during the transition.

- In principle, then, we can use v_0, v_1 and v_T to assess welfare consequences of reforms.
- But utility is an ordinal concept that we cannot quantify.

- To get around we can compute a consumption equivalent variation. To do so consider the optimal consumption allocation in the initial stationary equilibrium $\{c_s\}_{s=0}^{\infty}$, a CRRA utility and the associated $v_0(a, y)$ is

$$v_0(a, y) = E_0 \sum_{s=0}^{\infty} \frac{c_t^{1-\sigma}}{1-\sigma}$$

- Suppose we increase consumption in each date by a fraction g so that the new allocation is $\{(1+g)c_s\}_{s=0}^{\infty}$. The lifetime utility from that consumption allocation is:

$$
\begin{aligned}
v_0(a,y;g) &= E_0 \sum_{s=0}^{\infty} \frac{[(1+g)c_s]^{1-\sigma}}{1-\sigma} \\
&= (1+g)^{1-\sigma} E_0 \sum_{s=0}^{\infty} \frac{c_s^{1-\sigma}}{1-\sigma} \\
&= (1+g)^{1-\sigma} v_0(a,y)
\end{aligned}
$$

where

$$v_0(a,y;g=0) = v_0(a,y).$$

- If we want to quantify the welfare consequences of the policy reform for agent (a, y) we can ask: by what percent g do we have to increase consumption in the old stationary equilibrium in each date and state for the agent to be indifferent between living in the old stationary equilibrium and living through the transition induced by the policy reform.

- This percent g solves

$$v_0(a, y; g) = v_1(a, y)$$

or, rearranging:

$$\begin{aligned} (1+g)^{1-\sigma} v_0(a, y) &= v_1(a, y) \\ g(a, y) &= \left[\frac{v_1(a, y)}{v_0(a, y)}\right]^{\frac{1}{1-\sigma}} - 1 \end{aligned}$$

- If $g(a, y)$ is bigger than zero agents will benefit from the reform and $g(a, y)$ measures how much in consumption terms.

- Note that $g(a, y)$ depends on a and y. In the event of a tax increase on capital income one would expect households with a lot of assets lose badly, while households with little assets may even gain (taxes are lump-sum redistributed).

- Note that we only need to know $v_0(a, y)$ and $v_1(a, y)$ to compute $g(a, y)$, but our computation of the transition path gives us already v_1.

- Often studies ignore—with no argument for it: laziness is NOT an argument!—the transition and assess the steady state welfare consequences of policy reform.
  - ▷ To do so one computes

$$g_{ss}(a,y) = \left[\frac{v_T(a,y)}{v_0(a,y)}\right]^{\frac{1}{1-\sigma}}$$

- This is interpreted as steady-state welfare gain of an agent being born with characteristics (a, y).
- Steady state comparisons ignore the transition with possibly quantifiable consequences. For example, an increase in capital tax induces in principle a lower aggregate capital, hence consumption and a loss of welfare. However, along the transition path part of the capital stock is being eaten with the associated consumption and welfare derived from it.
- Whenever possible, avoid the parallel universe comparisons.

- We can also check the welfare consequences of a policy reform before households characteristics are revealed. At the steady state this is:
  - ▷ To do so one can compute

$$g_{ss} = \left[\frac{\int v_T(a,y) d\Phi_T}{\int v_0(a,y) d\Phi_0} \right]^{\frac{1}{1-\sigma}}$$

- We interpret this name as the welfare gain of an agent being born with characteristics (a, y) where $\int v_T(a,y)d\Phi_T$ is the expected lifetime utility of an agent in the new steady state, before the agent knows his pair (a, y)—behind the veil of ignorance (John Rawls). $\int v_0(a,y)d\Phi_0$ is defined accordingly.