Growth and Development Economics
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Homework 6, due Wed March 15 at 1.00pm

Question 1. A Two-Period Model of Accumulation and Migration with Uncertainty

Discuss all your results in each and all the following items:

1. **Solve the following urban problem:** This is a partial equilibrium 2-period model with a labor income endowment in the first and second period and the possibility to save/borrow up to a limit. Tomorrow's labor income is uncertain.

   Households maximize,

$$V(y_0) = \max_{\{c_0, c_1, a \geq \underline{a}\}} u(c_0) + \beta E u(c_1)$$

   subject to the following sequence of budget constraints,

$$c_0 + a = y_0$$
$$c_1 = \widetilde{y}_1 + (1+r)a$$

   where $E(\widetilde{y}_1) = \overline{y} + \varepsilon$, with $E(\varepsilon) = 0$ and $\frac{1}{2}$ probability for ε and $\frac{1}{2}$ probability for $-\varepsilon$.[1]

   Remark. Note that tomorrow's consumption must be positive, $c_1 \geq 0$, so $\widetilde{y}_1 \geq -(1+r)a$, which implies a lower bound (natural borrowing limit) of $\underline{a} \geq -\frac{y_{1,L}}{1+r}$. At the same time,

[1] When the constraint is binding, the solution is straightforward. When the constraint is not binding note that plugging the budget constraints into the objective function, households solve:

$$\max_{\{a\}} u(y_0 - a) + \beta \left[\frac{1}{2} u(y_1 + \varepsilon + (1+r)a) + \frac{1}{2} u(y_1 - \varepsilon + (1+r)a) \right]$$

The first order condition of a (i.e., FOC(a)) is,

$$u'(c_0) = \beta(1+r) E u'(c_1)$$
$$u'(y_0 - a) = \beta(1+r) \left[\frac{1}{2} u'(y_1 + \varepsilon + (1+r)a) + \frac{1}{2} u'(y_1 - \varepsilon + (1+r)a) \right]$$

which together with the consolidated budget constraint conform the set of equilibrium conditions.

Assuming CRRA utility function, the equilibrium conditions are:

$$(y_0 - a)^{-\eta} = \beta(1+r) \left[\frac{1}{2} (y_1 + \varepsilon + (1+r)a)^{-\eta} + \frac{1}{2} (y_1 - \varepsilon + (1+r)a)^{-\eta} \right]$$

We solve this nonlinear equation to find a^*.

After finding a, we can find c_0 using the first period budget constraint $c_0 = y_0 - a$. Note that a and c_0 are uniquely determined. Finally, can find c_1 using the second period budget constraint, and note that c_1 is not unique consumption today must be positive $c_0 \geq 0$, so $a \leq y_0$, which implies an upper bound for savings given by today's income/endowment. That is, the grid for feasible assets is $a \in \left[-\frac{y_{1,L}}{1+r}, y_0\right]$.

Assume the following parameter values:

parameter	value
β	.9
r	.04
η	2.00
y_1	.1
ε	.04
$\underline{a}$	$-\frac{y_{1,L}}{2(1+r)}$
$\underline{c}_0$	0.01
$\underline{c}_1$	0.03

To report your solutions

(a) Plot three graphs: c_0, c_1, and a with respect to an initial asset y_0 for CRRA utility: $u(c) = \frac{c^{1-\eta}}{1-\eta}$. Also for c_1, use a solid line for a good shock and dotted line for a bad shock.

(b) Add a line to the same graphs with the solution for the quadratic utility: $u(c) = -\frac{1}{2}(c-\bar{c})^2$. Set $\bar{c} = 0$.

2. **Solve the following rural problem.** The value of lving in rural areas differs from urban areas in that tomorrows consumption is defined by production and the selling of productive assets. There is also more risk associated with the farming technology than the urban labor income:

$$V = \max_{\{c_0, c_1, a \geq \underline{a}\}} u(c_0) + E\beta u(c_1)$$

subject to

$$c_0 + a + k = y_0$$
$$c_1 = (1 + \widetilde{z}k^{\alpha-1})k + (1+r)a$$

where $\widetilde{z}k^{\alpha}$ with $\alpha < 1$ is a decreasing returns to scale technology that generates production. That is, consumption tomorrow is the production that we get from capital (zk^{α}) plus its

as it depends on the realization of ε.

$$c_0^* = y_0 - a^*$$
$$c_{1,H} = y_1 + \varepsilon + (1+r)a^*$$
$$c_{1,L} = y_1 - \varepsilon + (1+r)a^*$$
$$s^* = \frac{a^*}{y_0}$$

sale k. Note that we can rewrite $c_1 = ak + zk^{\alpha} = k + zk^{\alpha-1}k = k + MP_k k = (1 + MP_k)k$, where MP_k is the marginal product of a or the (endogenous) rate of return on a, $r(k) = MP_k = zk^{\alpha-1}$.

Use the following parameter values:

parameter	value
β	0.9
r	0.04
η	2.00
ε	0.3
α	0.6
$\overline{a}$	-0.05

To report your solutions

(a) Plot three graphs: c_0, c_1, and a with respect to an initial asset y_0 for CRRA utility: $u(c) = \frac{c^{1-\eta}}{1-\eta}$. Also for c_1, use a solid line for a good shock and dotted line for a bad shock.

(b) Add a line to the same graphs with the solution for the quadratic utility: $u(c) = -\frac{1}{2}(c - \bar{c})^2$. Set $\bar{c} = 0$.

3. **Sove for the Migration Decision.** Find which agents decide to live in rural or urban areas ex-ante and ex-post of the realization of their type (i.e., initial wealth). Discuss your migratin results.

Question 2. A Heterogeneous Agent Structural Transformation (HAST) Economy

1. Write the household (and firms) problem for your economy in Question 1 extended with an overlapping generations structure for agents that enter the dynamic problem at age 18, live for 65 years, and decide to permanently migrate or not from rural to urban areas (or from urban to rural areas) at age 20. Add all the elements that you think are necessary to set up a well defined problem. Your agents will be heterogenous in assets, income, age, and area of residency (rural vs. urban). This HAST economy must be in general equilibrium.

2. Define the recursive competitive equilibrium (RCE) of your HAST economy.

3. Provide a solution algorithm to solve for your RCE.