

**Growth and Development Economics**  
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Homework 5, due Wed March 1 at 1.00pm

**Question 1. Factor Misallocation**

Discuss all your results in each and all the following items:

1. Individual output, capital and productivity are, respectively,  $y_i$ ,  $k_i$  and  $z_i$ . Assume that  $\ln z_i$  and  $\ln k_i$  follow a joint normal distribution. Assume that the correlation between  $z_i$  and  $k_i$  is zero, the variance of  $\ln z_i$  is twice larger than the variance of  $\ln k_i$ , and that average  $z$  and  $k$  is equal to one. Then simulate 10,000 observations and plot the joint density in logs and in levels.
2. Compute firm output  $y_i$  for each of your observations as:

$$y_i = z_i k_i^\gamma$$

where  $\gamma = 0.5$ . From now on consider your simulated  $z_i$ ,  $k_i$  and  $y_i$  as actual data.

3. Solve the following maximization problem:

$$Y^* = \max_{k_i} \sum_i z_i k_i^\gamma$$

subject to  $K = \sum_i k_i$ . To solve this problem use  $z_i$  from your actual data.

4. Compare your the optimal allocations  $k^*$  against the data.
5. Compute the output gains from reallocation, i.e.,  $\frac{Y}{Y^*}$  where  $Y = \sum_i y_i$  from the data.
6. Redo items (2)-(5) for  $\gamma = 0.8$ . Discuss your results.
7. Redo items (1)-(6) assuming that the correlation between  $z_i$  and  $k_i$  is 0.8.
8. Redo items (1)-(6) assuming that the  $z_i$  and  $k_i$  follow Pareto distributions. Assume that the scale parameter in the Pareto for  $z_i$  is high and that for  $k_i$  is low. That is, there is more inequality in  $z_i$  than in  $k_i$ .
9. Redo item (8) for different scale parameters in the Pareto distribution.