Complete Markets Tests

Growth and Development

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- Risk is at the core of decision making in poor countries.

- Welfare depends on the ability to cope with that risk.

- Ex-ante (i.e. diversification) and/or ex-post (i.e., access to credit) insurance mechanisms might be available.

Full Insurance Allocations

First things first. Here we provide the complete markets allocations and associated tests. We do this over time and over the life cycle.

A social planner solves:

$$\sum_i \omega_i \sum_{t=0}^{\infty} \beta_i^t \sum_{s^t} \pi(s^t) U((c_i(s^t), b_i(s^t))$$

subject to

$$\sum_i c_i(s^t) = \sum_i y_i(s^t)$$

and there is one such constraint per period.
Lagrangian,

$$\sum_i \omega_i \sum_{t=0}^{\infty} \beta_i^t \sum_{s^t} \pi(s^t) U((c_i(s^t), b_i(s^t)) - \mu_t \left[\sum_i c_i(s^t) - \sum_i y_i(s^t) \right]$$

First order condition of consumption for individual i at period t,

$$\omega_i \beta_i^t \pi(s^t) U_{c^i}((c_i(s^t), b_i(s^t)) = \mu_t \tag{1}$$

Hence,

$$\frac{U_{c^i}((c_i(s^t), b_i(s^t))}{U_{c^j}((c_j(s^t), b_j(s^t))} = \frac{\omega_j}{\omega_i}\left(\frac{\beta_i}{\beta_j}\right)^t$$

Result 1. *If $\beta_i = \beta \; \forall i$, full-risk sharing implies that the ratio of marginal utilities of consumption across individuals remains constant over time and states of the world. If $\beta_i \neq \beta \; \forall i$, full-risk sharing implies that the ratio of marginal utilities grows at a rate* $\ln \frac{\beta_i}{\beta_j}$

Taking logs of (10) on both sides, we find that:

$$\ln \omega_i + \ln \beta_i^t + \ln \pi(s^t) + \ln U_{c^i}((c_i(s^t), b_i(s^t)) = \ln \mu_t \tag{2}$$

Aggregating over all individuals i, we find that:

$$\sum_i \left(\ln \omega_i + \ln \beta_i^t + \ln \pi(s^t) + \ln U_{c^i}((c_i(s^t), b_i(s^t))\right) = \sum_i \ln \mu_t = N\mu_t$$

$$\frac{1}{N}\sum_i \ln \omega_i + \frac{1}{N}\sum_i \ln \beta_i^t + \frac{1}{N}\sum_i \ln \pi(s^t) + \frac{1}{N}\sum_i \ln U_{c^i}((c_i(s^t), b_i(s^t)) = \ln \mu_t \tag{3}$$

where $\sum_i = N$.

Then, equating RHS of (2) and (3) we find that:

$$\ln U_{c^i}((c_i(s^t), b_i(s^t)) - \frac{1}{N}\sum_i \ln U_{c^i}((c_i(s^t), b_i(s^t)) = -\left[\ln \omega_i - \frac{1}{N}\sum_i \ln \omega_i\right] - \left[\ln \beta_i^t - \frac{1}{N}\sum_i \ln \beta_i^t\right]$$

where we have used the fact that $\left[\pi(s^t) - \frac{1}{N}\sum_i \ln \pi(s^t)\right] = 0$. Further, if $\beta_i = \beta \; \forall i$, then

$$\ln U_{c^i}((c_i(s^t), b_i(s^t)) - \frac{1}{N}\sum_i \ln U_{c^i}((c_i(s^t), b_i(s^t)) = -\left[\ln \omega_i - \frac{1}{N}\sum_i \ln \omega_i\right] \quad (4)$$

and this equation (4) is the one almost invariable used to empirically test for market completeness.

Case 1: Exponential Utility

Assume that the felicity function is,

$$U((c_i(s^t), b_i(s^t)) = -\frac{1}{\eta} e^{b_i(s^t)} e^{-\eta c_i(s^t)} \tag{5}$$

then, marginal utility is

$$U_{c^i}((c_i(s^t), b_i(s^t)) = e^{b_i(s^t)} e^{-\eta c_i(s^t)}$$

and (4) becomes

$$\ln e^{b_i(s^t)} e^{-\eta c_i(s^t)} - \frac{1}{N} \sum_i \ln e^{b_i(s^t)} e^{-\eta c_i(s^t)} = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right]$$

$$b_i(s^t) - \eta c_i(s^t) - \frac{1}{N} \sum_i \left[b_i(s^t) - \eta c_i(s^t)\right] = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right]$$

$$-\eta c_i(s^t) = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right] - \left[b_i(s^t) - \frac{1}{N} \sum_i b_i(s^t)\right] - \eta \frac{1}{N} \sum_i c_i(s^t)$$

hence,

$$c_i(s^t) = \frac{1}{\eta}\left[\ln\omega_i - \frac{1}{N}\sum_i \ln\omega_i\right] + \frac{1}{\eta}\left[b_i(s^t) - \frac{1}{N}\sum_i b_i(s^t)\right] + \frac{1}{N}\sum_i c_i(s^t)$$

or

$$c_i(s^t) = \frac{1}{\eta}\left[\ln\omega_i - \frac{1}{N}\sum_i \ln\omega_i\right] + \frac{1}{\eta}\left[b_i(s^t) - \overline{b}(s^t)\right] + \overline{c}(s^t)$$

where $\overline{b}(s^t) = \frac{1}{N}\sum_i b_i(s^t)$ and $\overline{c}(s^t) = \frac{1}{N}\sum_i c_i(s^t)$.

Then, substracting previous year's consumption, we find

$$c_i(s^t) - c_i(s^{t-1}) = \frac{1}{\eta}\left[(b_i(s^t) - \overline{b}(s^t)) - (b_i(s^{t-1}) - \overline{b}(s^{t-1}))\right] + \overline{c}(s^t) - \overline{c}(s^{t-1}).$$

or

$$\Delta c_i(s^t) = \frac{1}{\eta}\Delta(b_i(s^t) - \overline{b}(s^t)) + \Delta\overline{c}(s^t) \tag{6}$$

Case 2: CRRA Utility

Assume that the felicity function is,

$$U((c_i(s^t), b_i(s^t)) = e^{b_i(s^t)} \frac{c(s^t)^{1-\sigma}}{1-\sigma} \tag{7}$$

then, marginal utility is

$$U_{c^i}((c_i(s^t), b_i(s^t)) = e^{b_i(s^t)} c(s^t)^{-\sigma}$$

and (4) becomes

$$\ln e^{b_i(s^t)} c(s^t)^{-\sigma} - \frac{1}{N} \sum_i \ln e^{b_i(s^t)} c(s^t)^{-\sigma} = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right]$$

$$b_i(s^t) - \sigma \ln c_i(s^t) - \frac{1}{N} \sum_i \left[b_i(s^t) - \sigma \ln c_i(s^t)\right] = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right]$$

$$-\sigma \ln c_i(s^t) = -\left[\ln \omega_i - \frac{1}{N} \sum_i \ln \omega_i\right] - \left[b_i(s^t) - \frac{1}{N} \sum_i b_i(s^t)\right] - \sigma \frac{1}{N} \sum_i \ln c_i(s^t)$$

hence,

$$\ln c_i(s^t) = \frac{1}{\sigma}\left[\ln\omega_i - \frac{1}{N}\sum_i \ln\omega_i\right] + \frac{1}{\sigma}\left[b_i(s^t) - \frac{1}{N}\sum_i b_i(s^t)\right] + \frac{1}{N}\sum_i \ln c_i(s^t)$$

or

$$\ln c_i(s^t) = \frac{1}{\sigma}\left[\ln\omega_i - \frac{1}{N}\sum_i \ln\omega_i\right] + \frac{1}{\sigma}\left[b_i(s^t) - \overline{b}(s^t)\right] + \overline{\ln c}(s^t)$$

where $\overline{b}(s^t) = \frac{1}{N}\sum_i b_i(s^t)$ and $\overline{\ln c}(s^t) = \frac{1}{N}\sum_i \ln c_i(s^t)$.

Then, substracting previous year's consumption, we find

$$\ln c_i(s^t) - \ln c_i(s^{t-1}) = \frac{1}{\sigma}\left[(b_i(s^t) - \overline{b}(s^t)) - (b_i(s^{t-1}) - \overline{b}(s^{t-1}))\right] + \overline{\ln c}(s^t) - \overline{\ln c}$$

or

$$\Delta \ln c_i(s^t) = \frac{1}{\sigma}\Delta(b_i(s^t) - \overline{b}(s^t)) + \Delta\overline{\ln c}(s^t) \quad (8)$$

Empirically...

- Using panel data, and assuming a CRRA utility function, we can compute household-specific changes over time (not necessarily adjacent periods) to to directly test the hypothesis of foull risk-sharing in (8) as follows,

$$\Delta \ln c_i(s^t) = \gamma \Delta \overline{\ln c}(s^t) + \phi_y \Delta \ln y_i(s^t), \tag{9}$$

where theory imposes that $\gamma = 1$ and $\phi_y = 0$.

Theory imposes that the individual consumption must follow aggregate consumption and not any other indivdual characteristic. That is, the paremeter estimate ϕ on any variable (e.g., income, assets, household composition...) other than aggregate consumption must be zero.

The case for exponential utility is analogous.

A variance-based test

Recall that the first order condition of consumption for individual i at period t,

$$\omega_i \beta_i^t \pi(s^t) U_{c^i}((c_i(s^t), b_i(s^t)) = \mu_t$$

Then, taking logs, aggregating over all individuals i, substracting, and assuming that the felicity function is $U((c_i(s^t), b_i(s^t)) = \frac{\left(\frac{c_i(s^t)}{b_i(s^t)}\right)^{1-\eta}}{1-\eta}$ we can write the FOC as

$$\ln \widetilde{c}_i(s^t) = \frac{1}{\eta} \ln \widehat{\omega}_i + \overline{\ln c}(s^t) \tag{10}$$

where $\widetilde{c}_i(s^t) = \frac{c_i}{b_i}(s^t)$ is adult-equivalent consumption, and hat-variables represent log-deviations from mean, $\ln \widehat{x}_i = \ln x_i - \frac{1}{N}\sum_i \ln x_i$, and $\overline{\ln c}(s^t) = \frac{1}{N}\sum_i \ln c_i(s^t)$ is aggregate (average) logged consumption.

It is interesting to note that we can use (10) to compute the change in the cross-sectional variance of adult-equivalent consumption $\widetilde{c}_i(s^t) = \frac{c_i}{b_i}(s^t)$ as,

$$Var\left(\ln \widetilde{c}_i(s^t)\right) = \frac{1}{\eta^2} Var\left(\ln \widetilde{\omega}_i\right)$$

because the cross-sectional variance of the aggregate consumption is zero. That is, the variance of (logged) consumption is the variance of the (logged) pareto weights.

This way, complete markets theory implies that the change in the cross-sectional variance of consumption over time must be zero, that is,

$$\Delta Var\left(\ln \widetilde{c}_i(s^t)\right) = 0. \tag{11}$$

because pareto weights do not change over time. As noted by Deaton and Paxon, Attanasio and others, with full-risk sharing the variance of consumption should not change with time.

We can empirically test this implication (11) for the cross-sectional variance with the following strategy

$$\Delta Var\left(\ln \widetilde{c}_i(s^t)\right) = \psi_y \Delta Var\left(\ln y_i(s^t)\right)$$

Complete market theory tells us that ψ_y must be zero.

We can interpret the parameter ψ_y as the degree transmission from income to consumption inequality, or more generally, how much consumption insurance an econmy has with respect to full insurance. This is useful for our purposes of comparing the degree of insurance separately across rural and urban areas (or across countries or time).

The same goes for the change in the variance of any other variable (e.g., assets).

So far, we have ignored the direct transmissions from transfers. But we can compute the transfers from urban to rural areas. And also check the degree of insurance with and without food transfers in both mean-based and variance-based tests.

Some literature

- Townsend (1994), Village India is well insured. This was a surprising result at that time.
- Udry (1995), AER Savings paper
- Udry (1994), Village Northern Nigeria
- Good review in Kinnan (2014).

Udry (1994), Village Northern Nigeria

- 4 villages in northern Nigeria.
- Informal credit. No written records.
- Borrower and lender negotiate the size of loan, but interest rates and repayment dates are almost never set.
- Widespread participation in 12 months survey: 75% of households lended and 65% borrowed. 50% of households were borrowers and lenders.
- 97% of loans (weighted by value) were between family/neighbors.
- 82% of lenders could account for the farming activities of their borrowers. Not much scope for imperfect information (no asymmetries).
- Only 3% of loans was backed by collateral.
- The fact that few loans go beyond villages suggests that information might play a role in sustaining these informal contracts. They are only sustainable where there is little imperfect info. Now, conditional on being in the same village, information does not seem to be an issue.

Udry (1995), AER Savings paper

When insurance markets are incomplete, saving and credit transactions assume a special role by allowing households to smooth their consumption streams in the face of random income fluctuations.

Question: Do households dissave when confronted with adverse shocks to their incomes?

- To explore this issue: use information on the receipt of (random) production shocks in 12 months (e.g. flooding). This direct measure of transitory shocks does not require the computation of residuals.
- He uses differences in assets (stocks) over time to interpret savings.
- Findings: Grain inventories grow more slowly upon the receipt of an adverse shock, but that livestock holdings are unaffected. Consistent with the smoothing consumption through buffer stocks.

Some results from "The Price of Growth: Consumption Insurance in China 1989-2009," with Yu Zheng (2016)

Table: Testing Full Risk Sharing: China and U.S.

		China 1989-2009			U.S. 1981-1984 (Mace (1991)		
		$\triangle \ln C$	$\triangle \ln y_i$	F-Test	$\triangle \ln C$	$\triangle \ln y_i$	F-Test
Nondurables	CRRA	.976	.066	213.81	.97	.04	22.69**
		[.948,1.003]	[.060,.072]		.07	.006	
	CARA	.976	.028	60.41	.99	.01	7.71**
		[.933,1.019]	[.023,.033]		.06	.003	
Durables	CRRA	.898	.099	21.07	1.00	-0.03	.39
		[.827,.969]	[.067,.131]		.06	.03	
	CARA	.704	.011	75.52	1.03	0.004	0.06
		[.632,.775]	[.009,.013]		.15	.02	
Total	CRRA	.970	.078	248.57	1.06	.04	14.12**
		[.939,1.002]	[.071,.085]		.08	.007	
	CARA	.913	.042	100.16	1.06	.03	1.27
		[.871,.955]	[.036,.048]		.11	.02	

Table: Testing Full Risk-Sharing: Pre- and Pos-1997

	Rural China				Urban China			
	Raw		Residual		Raw		Residual	
1989-2009:	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test
$\Delta \ln c_i$	.066 [.054,.078]	61.66	.064 [.052,.076]	56.26	.084 [.061,.107]	28.07	.091 [.067,.115]	27.79
1989-1997:	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test
$\Delta \ln c_i$	.041 [.024,.059]	11.79	.033 [.016,.050]	7.14	.108 [.069,.147]	14.84	.114 [.073,.155]	15.27
2000-2009:	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test	$\Delta \ln y_i$	F-Test
$\Delta \ln c_i$	.079 [.064,.094]	52.35	.079 [.064,.095]	50.41	.065 [.047,.102]	15.88	.081 [.052,.110]	15.39

Notes: We conduct the complete markets test for the rural and urban sample separately for the entire 1989 to 2009 period, the 1989 to 1997 period and the 2000 to 2009 period. In all cases in this table, we use the adult equivalent consumption and disposable income measures which we use in our benchmark estimation. The column title "Raw" refers to the fact that we use the (logged) adult equivalent consumption and disposable income in the tests, while the column title "Residual" refers to the fact that we use the residual (logged) consumption and disposable income in the tests. The residual (logged) consumption and disposable income are the residual components from a regression of (log) adult-equivalent consumption or income on dummies of sex, age, education, province and whether a minority, by areas of residence and by survey years.

Evidence from China suggests better consumption insurance in rural areas than in urban areas.

There are also substantial changes over time. After the 1997 change of polices in China (including transfers for urban areas in detriment of rural areas) the consumption insurance in urban areas improves with respect to that in rural areas.

What if there is no panel data?

- If there is no panel data, we can do complete markets tests by builiding synthetic-cohorts (see the work by Attanasio and Davis, Attanasio and Sevelenky)
- A synthetic-cohort g is treated as an individual. For example, the group g could include individuals of the same cohort and the same education level.

The full risk-sharing tests with synthetic cohorts g are analogous to those with panel data:

- **Means-Based Risk-Sharing Tests**

$$\Delta c_g(s^t) = \frac{1}{\eta}\Delta \widehat{b}_g(s^t) + \psi_y \Delta y_g(s^t) + \psi_a \Delta a_g(s^t) + \Delta \overline{\ln c}(s^t) \quad (12)$$

where $x_g = \frac{1}{N}\sum_{i\in g} \ln x_i$, and analogously for b.

- **Variance-Based Risk-Sharing Tests**

$$\Delta Var_g \left(\ln c_i(s^t)\right) = \theta \Delta Var_g \left(\ln b_i(s^t)\right) + \psi \Delta Var_g \left(\ln y_i(s^t)\right) \quad (13)$$

where $c_g = \frac{1}{N}\sum_{i\in g} c_i$, and analogously for b.